

# Principles For Verification Of Mathematical Fire Models

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**Abstract**—This article deals with methods of verification of mathematical fire models. The first part of the article is dedicated to introduce situation and basic conceptions in area of mathematical fire modelling. The biggest part of work is dedicated to explain methods of verification and validation. The aim of this article is a summary, identification of problems and proposal for improvement of verification of mathematical fire models.

**Keywords**—mathematical fire model, evaluation, verification, validation

## I. INTRODUCTION

The situation in the field of fire protection and construction is leading to the development of standards that regulate the level of safety better than traditional standards aimed at solving individual problems. With the development of computer technology using suitable for the creation of other sample standards be computer programs for fire modeling. These computer models can make comparisons between many factors and the team's required level of security.

Analytical models preparing to predict fire behavior began to develop in the 1960s 20th centuries. The efforts of scientists have been described by mathematical expressions of various phenomena that have been observed in the development and spread of fire. Various methods suitable for describing the course of a fire have been developed. Each of these models focuses on the assessment of partial manifestations and parameters. By unifying the models, it is possible to create a complex computer program that calculates the expected course of the fire based on the input parameters. When mathematical expressions of basic physical phenomena were developed, the original equations could be transformed into predictive equations for temperature, smoke and gas concentration, other required parameters, and then solved numerically. [1]

Fire modeling has been developing rapidly since the late 1980s. This development is mainly due to the great progress in the field of computer technology. Computer fire modeling programs are thus increasingly used in the field of fire safety and security engineering.

With the development and introduction of these models into the system of regulations, there must be gradual efforts to check their validity and verify the accuracy of the results. The accuracy of models solving individual phenomena should be addressed during development. However, iterations between different parts of the system are not always well understood.

Therefore, the methods needed to test complex model systems with large-scale tests and experiments are currently being adopted. [2]

## II. MATHEMATICAL MODELS OF FIRE AND ITS DISTRIBUTION

A mathematical model is an abstract model that is used in mathematical language to describe a system. A mathematical model is defined as a representation of the basic aspects of an existing system or the system we want to construct. Mathematical models can take many forms, such as dynamical systems, statistical models, differential equations, or theoretical models. These and other types of models can be overlaid to include different abstract structures.

To date, many computer models of fire have been developed. They are designed for different areas, difficulties, and purposes. Most models are based on basic laws of physics - the law determines matter, momentum, and energy. They mainly predict the fire created by the environment (mainly temperature) and the movement of smoke in enclosed spaces. In addition, some can predict fire resistance, the response of detectors and sprinklers. [3]

The most common are mathematical models of fire, which can be divided:

- probabilistic
  - statistical
  - network
- deterministic
  - zonal
  - field type

Mathematical models allow us to get a good prediction of some fire parameters. This is achieved by a combination of mathematical equations that describe physical phenomena. Because fires are constantly changing, the equations are substituted in the form of differential equations. The summary of equations can calculate the conditions created by a fire at a given time and a specific volume of air. The model assumptions that predicted the conditions within the control volume are always constant. So the control volume has the same temperature, smoke density, gas concentration, etc. Different models divide the building into different amounts of control volumes depending on the required degree of accuracy [4].

### III. EVALUATION AND VERIFICATION OF MODELS

The evaluation process is a very important part of the modeling process. It is needed to determine the acceptable use and limits of the fire model. Models will never work perfectly and will never be valid. What is required is an acceptable range of solutions. Which solution is considered acceptable is determined by the project guarantors, users, or, where appropriate, a third party. Efforts are currently being made to expand fire models outside fire laboratories in the future and to use them in design, fire brigades, and other organizations. Sufficient evaluation guarantees that their use will be proportionate to their scientific and technical basis, that the model chosen will be fit for purpose and that we can rely on the level of results. Thanks to this, we can avoid unwanted misuse [4].

The assessment of the model mainly includes the process of verification and validation of all its components. This process takes place from the very beginning of the program development, it is cyclical and repeatable (Fig. 1) [5]. The accuracy and validity of all subroutines are checked, but it is not possible to evaluate a program from only one of its components. For these reasons, the overall process of checking validity and accuracy always remains necessary to verify the whole model [6].

Verification is the provision of whether a matter is done correctly. Validation verifies that the right thing is done, in other words, that the results given by the model are valid. This is how the concepts of validation and verification are explained in the US, in the UK model creators and users use the definition of validation for verification and vice versa.

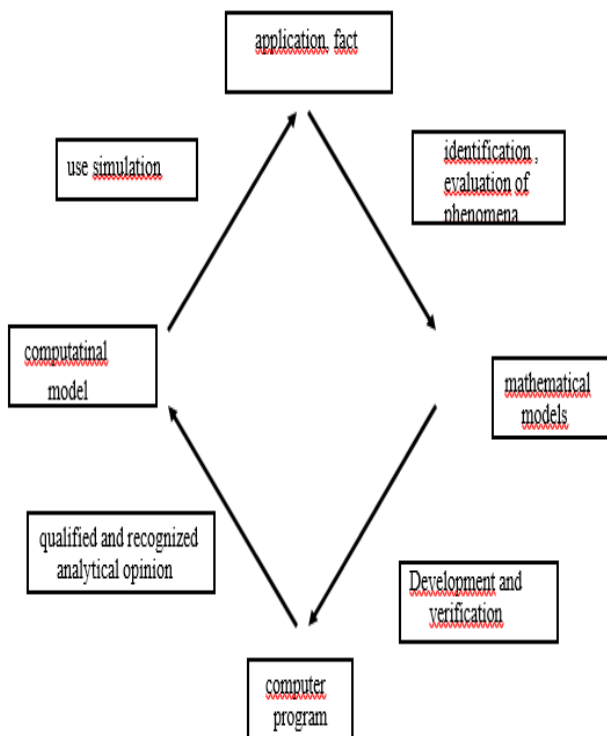


Figure 1: Schematic of the verification and validation process [5]

### IV. VERIFICATION

It relates to mathematical modeling using computers and is the domain of programmers. The purpose of verification is to verify that the program meets the specifications. In other words, the program, as it is written, accurately describes the model as it was designed. The complete modeling environment is checked, ie. sources, theory, assumptions, algorithms, code in terms of physical representation, and mathematical accuracy [7]. Checking their correct use must already be inside the program (eg it will display a warning when the scope is exceeded). Verification is not related to the properties of the relationships that created the model, but whether the recalculation of the relationships represented by the computer was done correctly.

Verification can take place in various ways, for example:

- static verification, which does not require the program to run and can therefore be performed at any stage of the program's development,
- dynamic validation, which derives program properties based on the results of running a program or prototype with selected inputs.

It is practically impossible to completely verify large complex computer programs, such as mathematical fire models. For this reason, some computer experts talk not about verification, but the degree of reliability of the program.

Successful verification or degree of reliability of the program is based on [8]:

- programmer qualification,
- mathematical model and method of solution,
- documented verification,
- the length of use of the program in practice,
- the diversity of uses of the program,
- current use.

### V. VALIDATION

The goal of validation is to verify that the program is meaningfully specified and that it provides a correct prediction for the input data set used. Tests the agreement between the behavior of the model and the real problem that is being modeled. The models were mostly obtained directly from specific experiments and were qualified according to them. The validity of This guarantee within a certain scope. So the validation of the program must be considered already during its creation. A program used outside its scope does not necessarily mean that it would be defective. [8]

The same problem applies to validation as for verification, ie the program cannot be valid. There is an effort to bring the model's predictions as close as possible to events that occur in real conditions. After applying all appropriate validation procedures, we will not obtain a valid model, but we will gain a good knowledge of all its strengths and weaknesses. We can then evaluate the severity of the simplifications adopted and say what changes have resulted in the result. Knowledge of the limits of applicability of the model's predictive abilities gives us due confidence in the results obtained. [9]

Validation depends on:

- aspects of the modeled real problem,
- the type of model used,
- the person requesting validation,
- a person interpreting the conclusions of the validation.

## VI. THE GENERAL METHODOLOGY OF FIRE MODEL CONTROL

The term model includes physical, mathematical, and numerical assumptions and approximations used to describe the combustion process and the movement of fumes, to describe the reaction of the object, persons, and fire safety equipment. This methodology assumes that the model is in the form of a computer program [10].

The following procedures are used to detect errors:

- assessment of the theoretical basis,
- source code check,
- analytical tests,
- empirical verification,
- comparison with other models.

### Type and use of the model

The model evaluation process addresses areas that are the focus of its users and creators. It is important what kind of model it is (zone model, field type model, model for special purposes) and how it will be used (design, fire reconstruction, litigation). The evaluation method used must be supported by both users and creators. The working group that carries out the evaluation consists of qualified and recognized experts not involved in the development of the program familiar with the issue. This group will prepare an independent study of the theoretical assumptions and mathematical procedures used in the model. This includes the introduction of experimental, statistical, and analytical techniques to address important model issues [10].

### Assessment of the theoretical basis

The theoretical basis of the model can be assessed by experts who are fully acquainted with the chemical and physical phenomena of fire. Evaluates the completeness of the documentation in terms of assumptions and approximations used. In addition, they should evaluate the accuracy of the data used for the constants and default values, and whether there is sufficient scientific evidence in the available scientific literature to support the procedures and assumptions used [11].

## VII. SOURCE CODE

It is important for the evaluation that the working code of the program is provided to the working group. However, this is not always possible, especially when it is a commercial program. If the source code is available, the program should be modified so that the code is available for review. The inspection should be performed by a third party and either manually or automatically. There are two ways to automatically check the source code. [11]

The first way is to use standard methods to control the program structure and interface. By these standard methods are meant programs that are contained directly in the model and perform an automatic check. These programs check the correctness of interfaces, undefined or ill-defined (used) variables and constants, and the completeness of cycles and threads. They do not check the correctness of the numerical use of constants or variables, but whether they are used correctly in the syntactic sense.

The second way is to run the program on different computer platforms and using different operating systems. A prerequisite for such control is the implementation of the programming language used on these platforms. [11]

### Analytical tests

If the problem has a mathematical solution, then they are a good way to verify the analytical test. The results obtained by the model can be verified by their mathematical solution. Usually, individual parts of the model, so-called sub-models, are analytically tested. However, there is not always an analytical solution, especially in the case of complex scenarios that are too complicated for such a method of verification [12].

## VIII. COMPARISON OF MODEL PREDICTIONS WITH EXPERIMENTAL RESULTS

User confidence in the model is increased when the user experience agrees with the model's predictions. This is based on the successful reconstruction of real fires or by comparing the predictions of the model with the data obtained experimentally. Comparing model predictions with experimental data is a very good way to evaluate the model (Fig. 2). Such verification ensures that errors in individual subroutines do not merge to make incorrect predictions. Program predictions should be made without reference to the experimental data used for comparison. Of course, this limitation does not include the necessary input data that could be obtained by testing or large-scale tests. Measurement uncertainties need to be taken into account. Model makers must not attempt to compare model results with empirical measurements [11].

## IX. COMPARISON OF THE MODEL WITH EXISTING PROGRAMS

The new model can be compared with models that have already been validated and created in practice. If the assessed model is correctly constructed, it should be performed by entering the same input data for the same result. Make a similar comparison select to reveal the weaknesses of the model, which are then debugged. For this comparison to be acceptable, the consequences must be demonstrated.

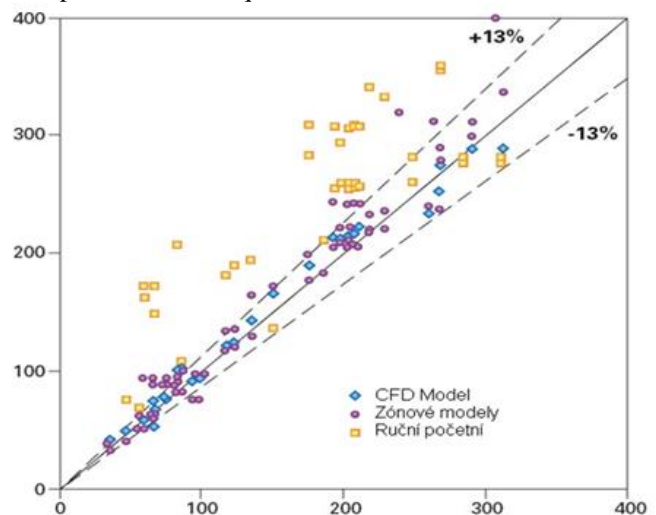


Figure 2 Measured vs. predicted temperature rise of the upper hot layer [3]

## X. CAUSES OF INAPPROPRIATE PROCEDURES AND INCORRECT ASSUMPTIONS

Using the above verification methods, inappropriate procedures and errors that occur in the model should be identified. (figure 3) These deficiencies may arise from the following causes [12]:

- use of inappropriate algorithms and physical relationships,
- use of wrong constants and default values,
- simplification of the phenomena described by omitting some of the processes accompanying the fire,
- poor numerical solution of systems of equations,
- errors in the program.

way	incorrect algorithm	incorrect constants	missing procedures	inappropriate numeric method	Code errors
theoretical study	•	•	•	•	
analytical tests			•		•
experimental verification		•	•		•
comparison with others programs	•	•	•		
source control program code					•

Figure 3 Ways of detecting errors and shortcomings of a mathematical model

#### XI. COMPARISON OF MATHEMATICAL MODELS WITH FIRE TESTS

Fire tests are divided into two categories: laboratory experiments and large-scale tests. Both of these categories provide us with a good level of comparison of whether the model matches real fires. Large-scale fire tests provide us with more qualitative results when the model mimics the actual fire conditions. (figure 4) With the help of laboratory experiments, we obtain detailed data which, when compared, emphasize the weaknesses of individual phenomena observed in the model. Fire tests are only relevant for the evaluation of a fire model if a sufficient number of them are performed. For a larger number of tests, different combinations will be tested when entering variables, the natural variability of the investigated phenomenon and the influence of uncertainties of experimental data will be manifested. Graphs that provide us with information without too much detail are a suitable means for evaluation and subsequent comparison of tests and models. For example, the graphs show the time course between test results and model-mediated results. Statistical methods are used to assess how significant differences and errors are. The level of agreement between the test and the model is usually described as "favorable", "acceptable", etc. [13]

For such fire tests to be used for comparison, they must be thoroughly prepared and performed. After successful completion of the fire test, the results and all observed phenomena shall be carefully documented so that they can be archived for later use. The results of fire tests are recorded in databases, which do not contain information on the accuracy of the measuring instruments used, etc. One such database is, for example, the FDMS from the National Institute of Standardization and Technology in the USA.



Figure 4 Large-scale fire test performed by NIST as part of a fire model validation project

#### XII. SUMMARY OF FINDINGS AND THEIR EVALUATION

The issue of verification of mathematical models of fire is very broad and includes knowledge from many scientific disciplines. This is because the phenomenon of fire itself is very complex and is influenced by many factors. Modelers try to take into account as many of these influences as possible in their programs so that the resulting simulation is as close as possible to the real environment. This requires close cooperation between experts from the fields of physics, chemistry, mathematics, computer science, fire protection, construction. The cooperation of these experts is necessary both in the creation of a program for mathematical modeling of fire and in its verification.

From the methods described above, it is clear that the verification process, which mainly includes the assessment of the theoretical basis, program code, and numerical procedures, is the domain of physicists, mathematicians, and programmers. On the contrary, the validation process, which compares the results of the model with large-scale fire tests, laboratory experiments, and other programs, is much more closely related to the practical side of the matter and is therefore also performed by experts in relevant fields such as research institutes and firefighters.

Verification of mathematical fire models cannot be understood as an isolated activity that follows only after the creation of the program but as an integral part of the entire development and subsequent use of the model. The verification methods described in this article are among the basic ones and are more or less common to all types of mathematical fire models. Certain differences they can be between the verification of zone models and field-type models, or between simpler and more complex models, respectively. Small perturbations of simple eigenvalues change parameters is a problem in general interest in applied mathematics. The purpose of this article is study the behavior of a simple eigenvalue of a singular number family of linear systems. [14]

The mathematical model is a virtual fire simulation, its numerical modeling is performed in several steps:

- input data input (default model conditions - equation variables, etc.),
- calculation of a system of mathematical equations,
- interpretation and evaluation of results.

Mathematical models can be further divided into deterministic models and models probabilistic. The basic difference between the different types of models is the consideration of uncertainty during modeling.

Deterministic models use no element of probability in their computational parts - coincidences. This means that if we enter the same input data, we must repeat the calculation to still get the same results. After entering the required inputs the process is visualized in parts. On the basis of the achieved level of visualization process we can assume a level of achieved quality of process. [15] Visualization recourse was described in more detail in Furthermore, deterministic models in fire protection are divided into zone models and field type models.

### XIII. CONCLUSION

Developments in the field of fire modeling using computer programs are getting faster and computer technology is being used in fire engineering on an ever-increasing scale. Introduction of fire models into the practice of specific works of designers and provide them with a means by which it will be possible to objectively compare results from different countries. As a result, we have adopted a benchmark for compliance and the team will reduce barriers to international trade in building materials, products, projects, and structures.

In order to be able to rely on the predicted results obtained by computer models, international and national standards must regulate the use of fire models in practice. It is necessary to establish general conditions for the verification of models that will be acceptable to developers and selected. Currently, the standards are already being worked on and it is only time that they will be implemented in our technical standards.

Mathematical modeling of fire using computer programs has been going on since about the 1980s, but it is still not very well integrated in practice. The key to the greater expansion of fire models is the development of verification methods that will guarantee that the model simulations and predictions are correct. This could be helped, for example, by greater dissemination of models outside universities and state institutions among private-sector designers. The involvement of these designers would certainly bring new information about the usability of the models in practice. Standards aimed at verifying mathematical models are now available. However, these standards provide only basic information on appropriate methods. It would be good if these standards were further developed to provide creators and users with more comprehensive information on verification procedures. Modeling is not just a national matter. The developed programs are used all over the world and experts from many countries cooperate in their verification. This international cooperation is very beneficial and should therefore be encouraged as much as possible.

The development and validation of models are closely linked to the development of computer technology. The more powerful computer technology we have at our disposal, the more demanding the models can be and the better the level of verification can be. This development has been very rapid in recent years, and this has caused complications in updating the standards for model validation.

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