Controller Design for Highly Maneuverable Aircraft Technology Using Structured Singular Value and Direct Search Method

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Abstract**—**The algebraic approach is applied to the HiMAT (Highly Maneuverable Aircraft Technology) control. The objective is to find a robust controller which guarantees robust stability and decoupled control of longitudinal model of a scaled remotely controlled vehicle version of the advanced fighter HiMAT. Control design is performed by decoupling the nominal multi-input multi-output system into two identical single-input single-output plants which are approximated by a 4th order transfer function. The algebraic approach is then used for pole placement design and the nominal closed-loop poles are tuned so that the peak of the μ-function is minimal. As an optimization tool, evolutionary algorithm Differential Migration is used in order to overcome the multimodality of the cost function yielding simple controller with decoupling for nominal plant which is compared with the *D-K* iteration through simulations of standard longitudinal manoeuvres documenting decoupled control obtained from algebraic approach for nominal plant as well as worst case perturbation.

I. Introduction

Algebraic methods are well known and easy to use for SISO (single-input single-output) systems described by continuous or discrete transfer functions. However, if applied to MIMO (multi-input multi-output) systems computational difficulties are increasing. In this paper, the problem of MIMO system design is treated via decoupling a MIMO system into two identical SISO plants which are then approximated by transfer functions with simple structure. This guarantees decoupled control for the nominal feedback loop and simplifies derivation of pole placement formulae.

In order to evaluate robust stability and performance the structured singular value denoted μ is used ([4] and [6]). The algebraic μ-synthesis [3] overcomes some difficulties connected with the *D-K* iteration as the final controller is not necessarily optimal with respect to structured singular value as the measure of robust performance [12]. Moreover, the controller obtained via the algebraic approach ([8] and [13]) can have simpler structure. This is due to the fact that there is no need of absorbance of the scaling matrices into the generalized plant, and hence, no need of further simplification causing deterioration of the frequency properties of the resulting controller.

The paper demonstrates an application of the algebraic μ-synthesis to the control of longitudinal model of a scaled remotely-piloted version of the advanced fighter HiMAT being a well known example of robust control design. The *D-K* iteration is used as a reference method and the results are compared through simulations for nominal and perturbed plants.

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The notation in the paper is as follows: $\mathbf{R}$ and $\mathbf{C}^{n\times m}$ are real numbers and complex matrices, respectively, $\|\cdot\|_\infty$ denotes norm in Hardy space of stable transfer matrices $\mathbf{H}_\infty(j\mathbf{R}, \mathbf{C}^{n\times m})$, $\mathbf{I}_{n\times m}$ is the unit matrix of dimension $n\times m$ and $\mathbf{R}_{PS}$ being the ring of Hurwitz-stable and proper rational transfer functions.

II. HiMAT Vehicle Model and Control Objectives

For the mathematical description and control objectives of the HiMAT see [1], [7], [9] and [11]. In the controller design, only longitudinal dynamics of the airplane will be considered which are supposed to be uncoupled from the lateral-directional ones. For the details on linearized models taking into account a set of flight conditions see [7]. The definition of state vector includes vehicle's basic rigid body quantities:

$$x^T = (\delta v,\ \alpha,\ q,\ \theta) \tag{1}$$

where δv is forward velocity, α is angle-of-attack, q is pitch rate and θ is pitch angle. The flight path angle (γ) is $\gamma = \theta - \alpha$. The motions in the vertical plane are defined using state variables:

δv - the velocity vector (forward speed)
α - the angle between the velocity vector and the aircraft's longitudinal axis (angle-of-attack)
q – the rate-of-change of the aircraft attitude angle (pitch rate)
θ - the aircraft attitude angle (pitch angle)

The actuator signals are the elevon command (δ_e) and canard command (δ_c). The measured quantities are α and θ.

There are three longitudinal manoeuvres making sense for the modelled problem:

- Change the altitude of the airplane with constant attitude and varying angle-of-attack representing the case where the attitude is constant and the velocity vector rotates (vertical translation).
- Change the attitude with constant flight path angle γ meaning that the velocity vector does not rotate (pitch pointing).
- Change the flight path angle with zero angle-of-attack implying standard airplane behaviour with no angle-of-attack alteration (direct lift).

III. Closed-Loop Feedback Structure

The controller design is performed using closed-loop feedback interconnection taking into account the uncertainty of the model and performance objectives in Fig. 1.

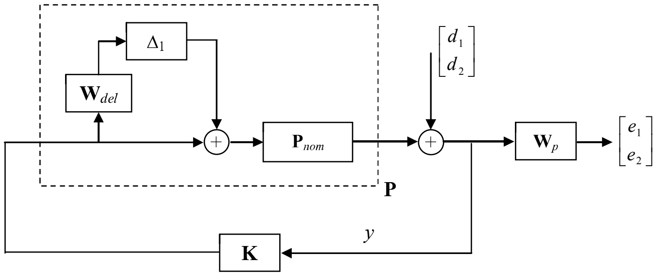

Figure 1. HiMAT closed-loop interconnection structure.

The dashed rectangle is the uncertain model of the airplane denoted **P**. Inside the rectangle are the nominal model of the airplane dynamics $\mathbf{P}_{nom}$ and two elements the weight $\mathbf{W}_{del}$ and unknown Δ_1, $\|\Delta_1\|_\infty < 1$ parameterizing the multiplicative uncertainty in the model.

Stabilizing controller **K** is designed so that the perturbed weighted sensitivity transfer function

$$\mathbf{S}(\Delta_1) \equiv \mathbf{W}_p[\mathbf{I} + \mathbf{P}_{nom}(\mathbf{I} + \Delta_1\mathbf{W}_{del})\mathbf{K}]^{-1} \tag{2}$$

has $\|\mathbf{S}(\Delta_1)\|_\infty < 1$ for $\|\Delta_1\|_\infty < 1$.

IV. Uncertainty Model

Nominal state-space model is defined by the following matrices

$$\mathbf{P}_{nom} = \left[\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \mathbf{C} & \mathbf{D} \end{array}\right] = \left[\begin{array}{cccc|cc} -0{,}023 & -37 & -19 & -32 & 0 & 0 \\ 0 & -1.9 & 0.98 & 0 & -0.41 & 0 \\ 0.012 & -12 & -2.6 & 0 & -78 & 22 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 57 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 57 & 0 & 0 \end{array}\right] \tag{3}$$

Given this nominal model HiMAT (i.e. $\mathbf{P}_{nom}$(s)) a stable 2×2 transfer matrix $\mathbf{W}_{del}(s)$ is specified representing the uncertainty weight. These two transfer matrices parameterize entire set of plants **P** which must be suitably controlled by the robust controller **K**:

$$\mathbf{P} \equiv \{\mathbf{P}_{nom}(\mathbf{I} + \Delta_1\mathbf{W}_{del}) \colon \Delta_1 \text{ stable}, \|\Delta_1\|_\infty \le 1\} \tag{4}$$

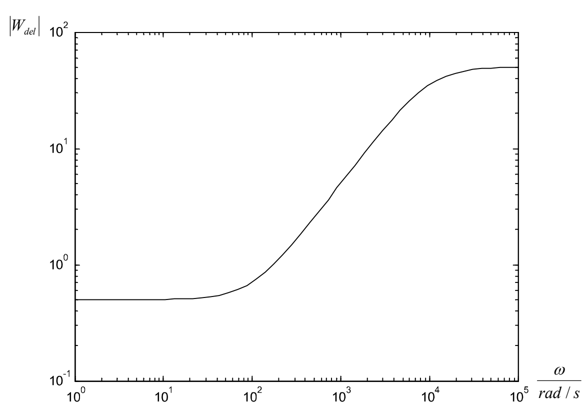

Figure 2. HiMAT multiplicative uncertainty weighting function.

Uncertainty in airplane modelling is captured in normalized unknown transfer function matrix Δ_1. The unknown transfer function $\Delta_1(s)$ is used to parameterize the potential differences between the nominal model $\mathbf{P}_{nom}(s)$ and the actual behaviour of the real airplane denoted **P**.

The uncertainty weight $\mathbf{W}_{del}$ has the form $\mathbf{W}_{del} \equiv W_{del}(s) \cdot \mathbf{I}_{2\times2}$ for a given scalar transfer function

$$W_{del}(s) = \frac{50(s+100)}{s+10000} \tag{5}$$

The set of plants represented by this uncertainty weight is

$$\mathbf{P} \equiv \left\{ \mathbf{P}_{nom}\left(\mathbf{I}_{2\times2} + \frac{50(s+100)}{s+10000}\Delta_1 \right) \colon \Delta_1 \text{ stable}, \|\Delta_1\|_\infty \le 1 \right\} \tag{6}$$

The bode plot of the uncertainty weight $W_{del}(s)$ is in Fig. 2.

V. Specifications of Closed-Loop Performance

The performance of the closed-loop system is evaluated using the output sensitivity transfer function $(\mathbf{I} + \mathbf{PK})^{-1}$. In this problem, a simple weight of the form $\mathbf{W}_p(s) = W_p(s)\,\mathbf{I}_{2\times2}$ is used with

$$W_p(s) = \frac{0.5(s+3)}{s+0.03} \tag{7}$$

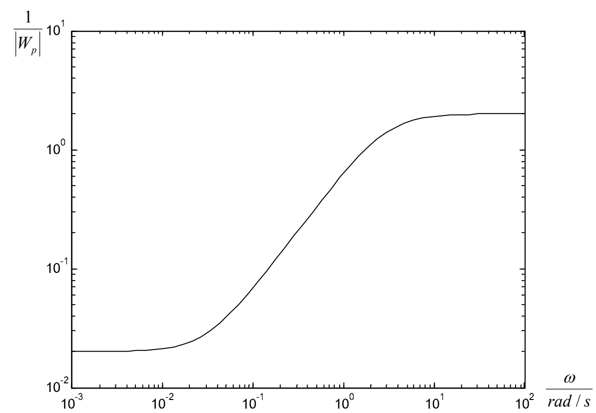

Figure 3. Inverse of the HiMAT performance weight.

Performance objective is achieved if $\|\mathbf{W}_p(\mathbf{I} + \mathbf{PK})^{-1}\|_\infty < 1$. That is, $\|\mathbf{W}_p(\mathbf{I} + \mathbf{PK})^{-1}\|_\infty < 1$ if and only if, at all frequencies, $\overline{\sigma}[(\mathbf{I} + \mathbf{PK})^{-1}(j\omega)] < |1/W_p(j\omega)|$. The bode plot of $1/W_p$ is shown in Fig. 3.

VI. Controller Design

Controller design via the structured singular value consists of making the open-loop interconnection and the controller design itself. In this paper, the *D-K* iteration and algebraic μ-synthesis is used. The results are verified by simulations.

A. D-K iteration

Performance specifications, model uncertainty and noise suppression requirements are incorporated into the LFT interconnection in Fig. 4. Here, the generalized plant $\mathbf{G}(s)$ is a transfer matrix with eight inputs and six outputs and $\mathbf{W}_n$ is a sensor noise weighting matrix in the form

$$\mathbf{W}_n(s) = W_n(s)\,\mathbf{I}_{2\times2} \tag{8}$$

with

$$W_n(s) = \frac{2s + 2.56}{s + 320} \qquad (9)$$

The weight W_n takes into account high frequency noise arising in sensors. With respect to this interconnection, define a block diagonal matrix

$$\mathbf{\Delta} \equiv \left\{ \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix} : \Delta_1 \in \mathbf{C}^{2\times 2}, \Delta_2 \in \mathbf{C}^{4\times 2} \right\} \subset \mathbf{C}^{6\times 4} \qquad (10)$$

The first block represents the uncertainty in the model of airplane. The second corresponds to sensor noise and performance weighting functions.

The structured singular value μ is given by the following definition:

Definition 1: For $\mathbf{M} \in \mathbf{C}^{4\times 6}$ is $\mu_{\mathbf{\Delta}}(\mathbf{M})$ defined as

$$\mu_{\mathbf{\Delta}}(\mathbf{M}) = \frac{1}{\min\{\overline{\sigma}(\Delta) : \Delta \in \mathbf{\Delta}, \det(\mathbf{I} - \mathbf{M}\Delta) = 0\}} \qquad (11)$$

If no such $\Delta \in \mathbf{\Delta}$ exists, for which $\mathbf{I} - \mathbf{M}\Delta$ is singular, then $\mu_{\mathbf{\Delta}}(\mathbf{M}) = 0$.

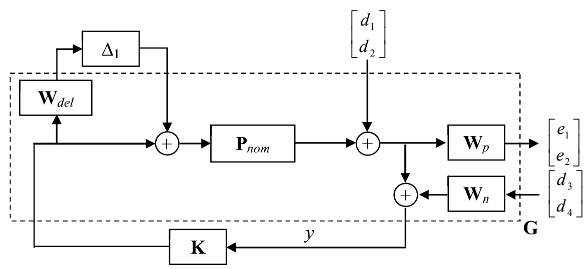

Figure 4. HiMAT open-loop interconnection structure.

Define $\mathbf{G}_P \equiv \mathbf{F}_u(\mathbf{G}, \Delta_1)$ and let $\mathbf{F}_l(\mathbf{G}, \mathbf{K})$ be stable then it can be proved [5] that, for all Δ_1 stable with $\|\Delta_1\|_\infty < 1$, performance condition $\|\mathbf{F}_l(\mathbf{G}_P, \mathbf{K})\|_\infty \leq 1$ hold and the closed loop in Fig. 5 is well posed and internally stable if and only if

$$\sup_{\omega \in \mathbf{R}} \mu_\Delta[\mathbf{F}_l(\mathbf{G}, \mathbf{K})] < 1 \qquad (12)$$

where $\mathbf{F}_u$ and $\mathbf{F}_l$ denotes upper and lower linear fractional transformation, respectively.

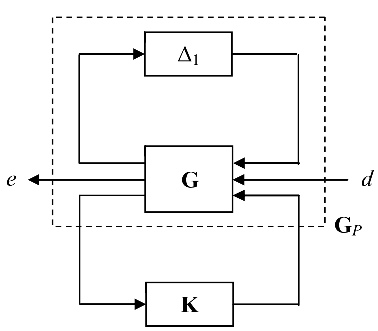

Figure 5. Scheme explaining robust performance condition.

Via the *D-K* iteration, a 30th order controller satisfying the robust performance condition (12) has been obtained (see Fig. 6). Step response for the vertical translation manoeuvre is not a monotonous function. However, if the plant is perturbed by the worst-case perturbation then there is not a significant deterioration in comparison with the nominal case (see Fig. 7 and 8).

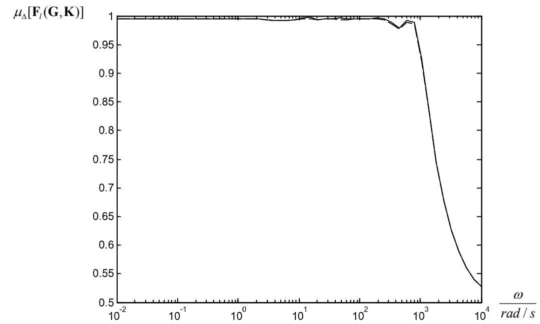

Figure 6. μ-plot for the *D-K* iteration controller.

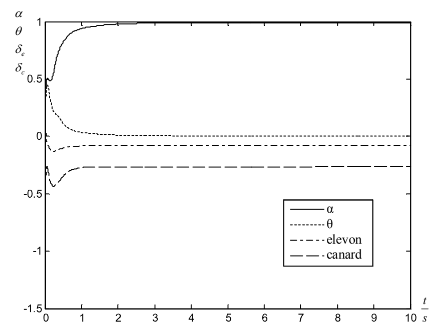

Figure 7. Step response for the nominal plant (*D-K* iteration).

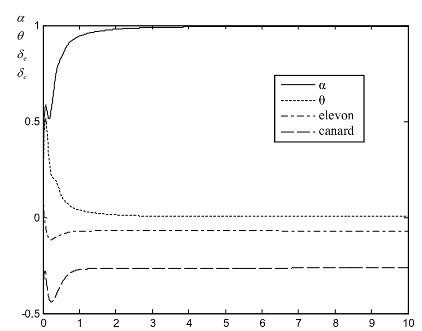

Figure 8. Step response for the worst case perturbation (*D-K* iteration).

B. Algebraic approach

For the purposes of algebraic μ-synthesis, the nominal MIMO system is decoupled into two identical SISO plants. The nominal HiMAT model can be defined in terms of transfer functions as:

$$\mathbf{P}_{nom}(s) \equiv \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \equiv \qquad (13)$$

$$\equiv \begin{bmatrix} \dfrac{-23.7s^3 - 4445.1s^2 - 106.0s - 9.4}{s^4 + 4.55s^3 + 16.83s^2 + 1.65s + 0.75} & \dfrac{1262s^2 + 29s}{s^4 + 4.55s^3 + 16.83s^2 + 1.65s + 0.75} \\ \dfrac{-4458s^2 - 8293s - 174}{s^4 + 4.55s^3 + 16.83s^2 + 1.65s + 0.75} & \dfrac{1228s^2 + 2468s + 55}{s^4 + 4.55s^3 + 16.83s^2 + 1.65s + 0.75} \end{bmatrix}$$

For decoupling of the nominal plant $\mathbf{P}_{nom}$, it is satisfactory to have the controller in the form

$$\mathbf{K}(s) = K(s)\det[\mathbf{P}_{nom}(s)]\frac{1}{P_{11}(s)}[\mathbf{P}_{nom}(s)]^{-1} \qquad (14)$$

The decoupling matrix incorporated into the controller does not cancel unstable poles or zeroes implying that internal stability of the nominal feedback loop is achieved. The multidimensional control is reduced into finding single-input single-output controller $K(s)$ obtained by tuning the poles of the nominal feedback loop with the nominal plant

$$\begin{aligned}\mathbf{P}_{dec}(s) &= \frac{1}{P_{11}(s)}\det[\mathbf{P}_{nom}(s)][\mathbf{P}_{nom}(s)]^{-1}\mathbf{P}_{nom}(s) \\ &= \frac{1}{P_{11}(s)}\det[\mathbf{P}_{nom}(s)]\mathbf{I}_{2\times 2}\end{aligned} \quad (15)$$

Define

$$P_{dec} \equiv \frac{1}{P_{11}(s)}\det[\mathbf{P}_{nom}(s)] \quad (16)$$

The transfer function P_{dec} is approximated by the 4$^{\text{th}}$ order system

$$P^{*}_{dec}(s) = \frac{b_{dec}(s)}{a_{dec}(s)} = \frac{-30448s - 688}{s^4 + 4.553s^3 + 16.833s^2 + 1.652s + 0.750} \quad (17)$$

and the controller $K = \dfrac{N}{M}$ is obtained by solving the Diophantine equation

$$A_{dec}M + B_{dec}N = 1 \quad (18)$$

with $A_{dec}, B_{dec}, M, N \in \mathbf{R}_{\text{ps}}$. It can be shown that the asymptotic tracking of the reference signal is achieved if and only if $A_{dec}M$ is divisible by F_r and the disturbance is suppressed if $A_{dec}M$ is divisible by F_d. Here, F_r and F_d denote Laplace transforms of the reference and disturbance, respectively. By the analysis of the polynomial degrees of a_{dec} and b_{dec} the transfer functions A_{dec}, B_{dec}, M and N are chosen so that the number of closed-loop poles is minimal and asymptotic tracking is achieved:

$$A_{dec} = \frac{a_{dec}}{\prod_{i=1}^{4}(s+\alpha_i)}, \quad B_{dec} = \frac{b_{dec}}{\prod_{i=1}^{4}(s+\alpha_i)} \quad (19)$$

$$M = \frac{sm}{\prod_{i=5}^{8}(s+\alpha_i)}, \quad N = \frac{n}{\prod_{i=5}^{8}(s+\alpha_i)} \quad (20)$$

The degrees of polynomials m and n are:

$$\partial m = 3, \partial n = 4 \quad (21)$$

Thus, the characteristic polynomial of the nominal closed loop has 8 poles $-\alpha_i$ representing the tuning parameters. The resulting controller K has the structure:

$$K(s) = \frac{n}{sm} \quad (22)$$

The open-loop interconnection is the same as for the D-K iteration but performance weight is relaxed:

$$W_p(s) = \frac{0.055(s+3)}{s+0.03} \quad (23)$$

The cost function is defined by

$$\sup_{\omega\in\mathbf{R}} \mu_{\Delta}[\mathbf{F}_l(\mathbf{G},\mathbf{K})] \quad (24)$$

Through Differential Migration (e.g. [2]) nominal closed loop poles were obtained:

$$\boldsymbol{\alpha} = [0.0095, 0.597, 0.001, 0.3975, 0.8038, 44.2131, 1.1358\cdot 10^9, 8.5867\cdot 10^{-4}] \quad (25)$$

and the scalar part of the controller:

$$K(s) = \frac{-3.18\cdot 10^5 s^4 - 1.68\cdot 10^6 s^3 - 2.98\cdot 10^6 s^2 - 1.65\cdot 10^6 s - 0.0015}{s^4 + 1.77\cdot 10^5 s^3 + 1.09\cdot 10^8 s^2 + 3.64\cdot 10^6 s} \quad (26)$$

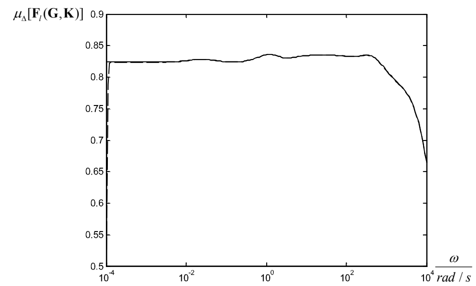

Figure 9. μ-plot for the algebraic approach.

The relaxed performance weight is justified by the additional postulate of the decoupled control for the nominal closed loop which is not present in the D-K iteration case making the task of achieving robust performance and stability more difficult. The relaxing of the performance weight does not degrade the uncertainty model and the resulting quality of the controller can be assessed by the simulation for the nominal and perturbed plant.

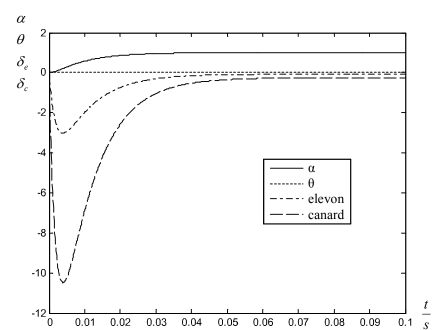

Figure 10. Step response for the nominal plant (alg. approach).

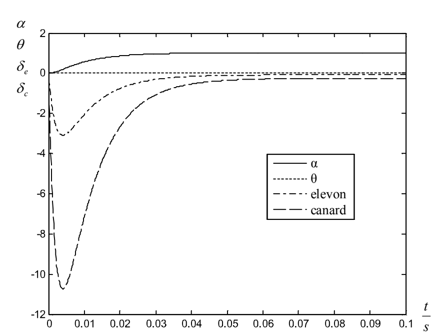

Figure 11. Step response for the worst case perturbation (alg. approach).

In order to overcome multimodality of (24), an evolutionary algorithm Differential Migration was used for searching the optimal values of α_i. Rough results obtained from DM were tuned up by the Nelder-Mead simplex method. The poles were constrained to the interval 0 to –20. The resulting controller has 24 states including decoupling part and satisfies condition (12) which guarantees robust performance and stability (see Fig. 9). Simulation of the vertical translation manoeuvre for the stepwise reference signal shows that the response is monotonous and significantly faster than for the *D-K* iteration (Fig. 10). Set point tracking for both the nominal and perturbed plant is achieved during 0.04 *s* compared to 4 *s* for the controller obtained by the *D-K* iteration. If the plant is perturbed by the worst-case perturbation then there is no significant deterioration in comparison with the nominal case (see Fig. 11). Moreover, the decoupling property is held for both cases.

VII. Conclusion

The paper has presented an application of the algebraic approach to the HiMAT airplane. The nominal MIMO plant was decoupled into two identical SISO systems and the controller was designed via optimization of the poles of the nominal closed loop. The performance and robustness were evaluated by the peak of the μ-function in frequency domain. Besides its simpler structure, the resulting controller satisfies the robust performance condition and guarantees the robust stability providing monotonous step response and significantly faster set point tracking during the vertical translation manoeuvre than the *D-K* iteration for both the nominal and perturbed plant. Although it was not intended during design, the decoupled control has been achieved also for the perturbed plant. The better performance of the controller obtained by the algebraic approach is due to the fact that the algebraic method implements the decoupled control for the nominal closed loop. This scheme can be implemented in the scope of the standard design using model matching bringing another step into the process of obtaining the controller (see [10]). The *D-K* iteration without decoupling makes a trade-off between robust stability and performance, however, the higher robustness is achieved at the expense of worse performance as it fully utilizes the MIMO structure of the controller.

Acknowledgment

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