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Application of the Robust Control Toolbox for Time Delay Systems with Parametric and Periodic Uncertainties Using SSV to Uncertain Time Delay System with Astatism

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Abstract: Application of the Robust Control Toolbox for Time Delay Systems with Parametric and Periodic Uncertainties Using SSV (Structured Singular Value) for the Matlab system to Uncertain Time Delay System with Astatism is performed. The D-K iteration and the algebraic approach implemented in the toolbox are applied to 2nd order system with astatism and uncertain time delay and two other parameters in the numerator and denominator of the plant transfer function. Multiplicative uncertainty is used for treating uncertain time delay, the parametric uncertainty is modelled using general interconnection for the systems with parametric uncertainty in numerator and denominator.

Keywords: Astatism; parametric uncertainty; structured singular value; global optimization; direct search methods

1. INTRODUCTION

Parametric uncertainties has been an issue of robust control for several decades, the first tool was Mapping Theorem Zadeh and Desoer (1963) succeeded by Kharitonov Theorem [Kharitonov (1978), Barmish (1984) and Bialas (1989)], Edge Theorem [Bartlett et al. (1988), Barmish (1989) and Sideris and de Gaston (1986)] and Generalized Kharitonov Theorem Chapellat and Bhattacharyya (1989) treating conservatism in applications to feedback loop with SISO (single-input single-output) controller. One of the latest results is tree structured decomposition Barmish et al. (1989) yielding a general procedure allowing the analysis of complex closed-loop characteristic polynomials in a polynomial time and the results for specific multilinear structures [Barmish and Shi (1990), Chapellat et al. (1993) and Fu et al. (1995)] considering the closed-loop characteristic polynomials corresponding to the series connections of interval plants.

In this paper, toolbox treating parametric and periodic uncertainties using structured singular value [SSV or μ, see Packard and Doyle (1993)] implementing both the algebraic approach with subsequent optimization using evolutionary algorithm [Dlapa (2018)] and *D-K* iteration as reference method is applied to 2nd order system with astatism and uncertain time delay and parameters. The toolbox solves both parametric and dynamic uncertainties including uncertain time delay. The Robust Control Design Toolbox for Time Delay Systems with Parametric and Periodic Uncertainties Using SSV (http://dlapa.cz/homeeng.htm) deals with uncertain time delay and parametric uncertainties in the numerator and denominator of the plant transfer function. The controller is derived for two-degree-of-freedom and single feedback loop [2DOF and 1DOF see Dlapa and Prokop (2014)].

The controller is tuned using pole placement of nominal closed loop poles solving Diophantine equation in the ring of Hurwitz-stable and proper rational functions ($\mathbf{R}_{PS}$). The poles of the nominal closed loop are tuned via direct search methods - Differential Migration Dlapa (2017) and Nelder-Mead simplex method managing the issue of multimodality of the structured singular value in relationship with nominal closed loop poles. This algorithm tackles impossibility of usage of the weights with poles on imaginary axis and convergence to a global or even local minimum causing non-optimality of the resulting controller in the *D-K* iteration Stein and Doyle (1991).

For reference, the controller derived using the *D-K* iteration [see Doyle (1985)] is compared to the one obtained from the algebraic approach showing the pros and cons of both procedures. The resulting controllers are compared in simulations of step response for different values of time delays and periodic changes of parameters with simple feedback loop and two-degree-of-freedom structure (1DOF and 2DOF).

Notation used in the paper: $\|\cdot\|_\infty$ is $\mathbf{H}_\infty$ norm, $\overline{\sigma}(\cdot)$ denotes maximum singular value, $\mathbf{R}$ and $\mathbf{C}^{n\times m}$ are real numbers and complex matrices, respectively, $\overline{\mathbf{R}}_+$ are positive real numbers, $\mathbf{I}_n$ is the unit matrix of dimension n and $\mathbf{R}_{PS}$ denotes the ring of Hurwitz-stable and proper rational functions.

2. DEFINITIONS

Define $\mathbf{\Delta}$ as a set of block diagonal matrices

$$\begin{aligned}\mathbf{\Delta} \equiv \{&\mathrm{diag}[\delta_1 I_{r_1},\ldots,\delta_S I_{r_S},\delta_1 I_{c_1},\ldots,\delta_T I_{c_T},\Delta_1,\ldots,\Delta_F,\Delta_1,\ldots,\Delta_K]:\\ &\delta_s \in \mathbf{C}, s=1\ldots S, \delta_t \in \mathbf{R}, t=1\ldots T, \Delta_f \in \mathbf{C}^{m_f\times m_f}, f=1\ldots F, \Delta_k \in \mathbf{R}^{n_k\times n_k}, k=1\ldots K\}\end{aligned} \tag{1}$$

where S, T is the number of repeated scalar complex and real blocks,
F, K is the number of full complex and real blocks,
$r_1,\ldots, r_S$, $r_1,\ldots, r_T$, $m_1,\ldots, m_F$, $n_1,\ldots, n_K$ are positive integers defining dimensions of scalar and full blocks.

For consistency among all the dimensions, the following condition must be held

$$\sum_{s=1}^{S} r_s + \sum_{t=1}^{T} m_t + \sum_{f=1}^{F} r_f + \sum_{k=1}^{K} m_k = n \tag{2}$$

Definition 1: For $\mathbf{M} \in \mathbf{C}^{n\times n}$ is $\mu_{\mathbf{\Delta}}(\mathbf{M})$ defined as

$$\mu_\Delta(\mathbf{M}) \equiv \frac{1}{\min\{\overline{\sigma}(\Delta) : \Delta \in \mathbf{\Delta}, \det(\mathbf{I} - \mathbf{M}\Delta) = 0\}} \tag{3}$$

If there is no $\Delta \in \mathbf{\Delta}$ making $\mathbf{I} - \mathbf{M}\Delta$ singular, then $\mu_{\mathbf{\Delta}}(\mathbf{M}) =$ $= 0$.

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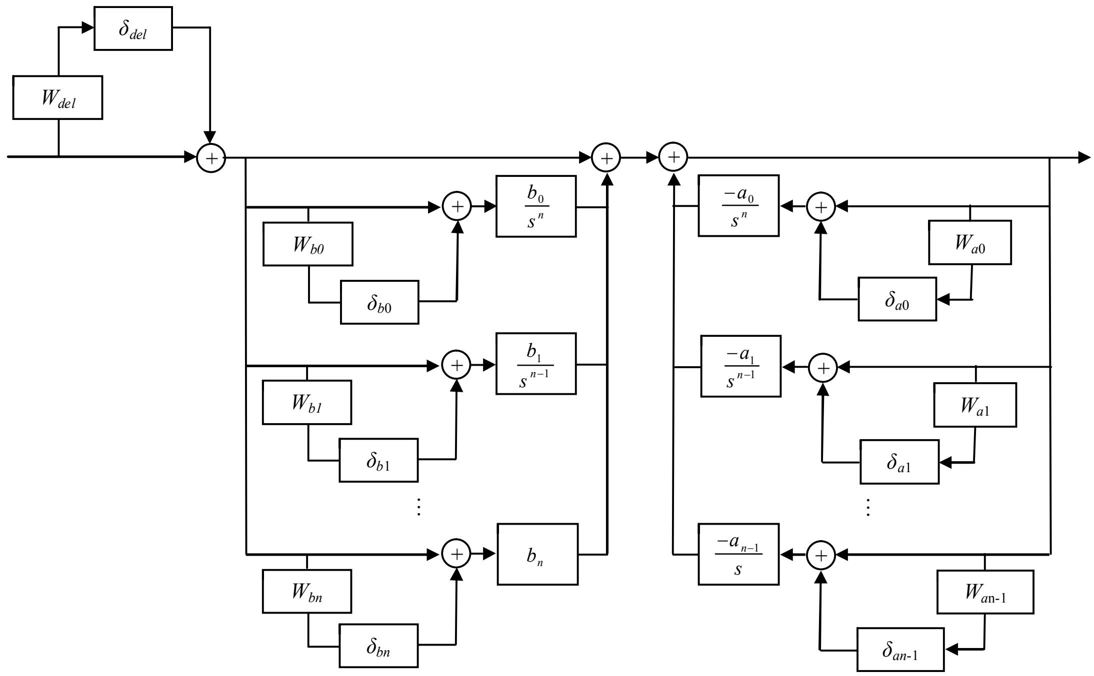

Fig. 1. Modelling general parametric uncertainties system

3. MODELLING OF PARAMETRIC UNCERTAINTIES FOR SSV DESIGN

Consider general system with uncertain numerator and denoinator and uncertain time delay treating parametric, periodic and time delay uncertainties:

$$P(s) \equiv \frac{(b_0 + b_1 s + \ldots + b_n s^n)e^{-\tau s}}{s^n + a_0 + a_1 s + \ldots + a_{n-1} s^{n-1}}$$

$a_i \in [\,\overline{a}_i - A_i,\ \overline{a}_i + A_i]$, $b_i \in [\,\overline{b}_i - B_i,\ \overline{b}_i + B_i]$,

$i = 0, 1, \ldots, n-1$, $\tau \in [0, T_d]$ (4)

Time delay and parametric uncertainties vary in the preefined intervals.

$$\left|\delta_{ai}\right| < 1 \ \left|\delta_{bi}\right| < 1 \ \left|\delta_{del}\right| < 1 \quad (5)$$

And for weights W_{ai}, W_{ai} and W_{del} the following inequalities must be held for all $\omega \in \mathbf{R}$:

$$W_{ai} = A_i, i = 0,1,\ldots,n-1 \quad (6)$$

$$W_{bi} = B_i, i = 0,1,\ldots,n \quad (7)$$

$$\left|W_{del}(j\omega)\right| > \left|1 - e^{j\omega T_d}\right| \quad (8)$$

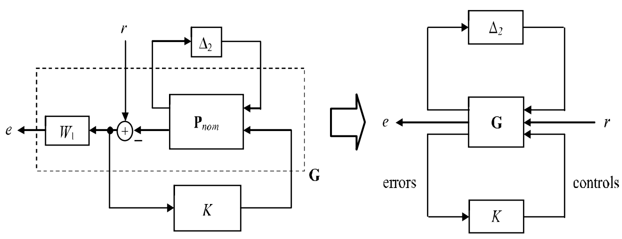

Fig. 2. Closed-loop interconnection

Plant (4) and Fig. 1 can be transformed to the interconnections in Fig. 2 with the sensitivity function as a performance indicator and $\mathbf{P}_{nom}$ being open-loop interconnection from Fig. 1.

Perturbation matrix has the form:

$$\Delta_2 \equiv \begin{bmatrix} \Delta_a & 0 & 0 \\ 0 & \Delta_b & 0 \\ 0 & 0 & \delta_{del} \end{bmatrix} \quad (9)$$

$$\Delta_a \equiv \begin{bmatrix} \delta_{a0} & 0 & \cdots & 0 \\ 0 & \delta_{a1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \delta_{an-1} \end{bmatrix} \quad (10)$$

$$\Delta_b \equiv \begin{bmatrix} \delta_{b0} & 0 & \cdots & 0 \\ 0 & \delta_{b1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \delta_{bn} \end{bmatrix} \quad (11)$$

For stability and performance Theorem 1 and the following Corollary 1 hold:

Theorem 1: For Δ defined by (9) the loop in Fig. 2 is well-posed, internally stable and $\left\|\mathbf{F}_L[\mathbf{F}_U(\mathbf{G},\Delta_2),K]\right\|_\infty \le 1$ if and only if

$$\sup_{\substack{\omega \in R \\ K \text{ stabilizing } \mathbf{G}}} \mu_\Delta[\mathbf{F}_L(\mathbf{G},K)(j\omega)] \le 1 \quad (12)$$

with $\mathbf{\Delta} \equiv \left\{ \begin{bmatrix} \delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix}, \delta_P \in \mathbf{C} \right\}$.

Proof: The proof is the same as in Doyle et al. (1982) and Packard and Doyle (1993) except for the fact that perturbations are complex matrices which simplifies the proof and complies with the definition of μ (Definition 1). ■

Define sensitivity function as transfer function from reference r to error e in Fig. 3:

$$S \equiv \frac{1}{1+PK} \tag{13}$$

Now, as a consequence of Theorem 1, a sufficient condition for the robust stability and performance of the feedback loop in Fig. 3 can be formed for sensitivity function S and family of plants (4).

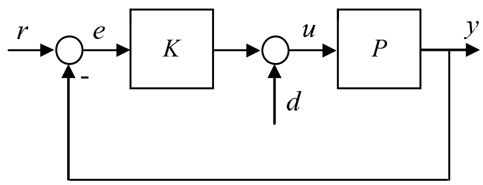

Fig. 3. Feedback loop

Corollary 1: For the set of plants (4), the feedback loop in Fig. 3 is internally stable and $\|SW_1\|_\infty \le 1$ if and only if (12), (5), (6), (7) and (8) hold.

Proof: The proof follows from Fig. 1, inequalities (12), (5), definitions (6), (7) and (8) and Theorem 1. ■

4. ALGEBRAIC μ-SYNTHESIS

The algebraic μ-synthesis can be applied to any control problem that can be transformed to the loop in Fig. 2 where $\mathbf{G}$ denotes the generalized plant, $\mathbf{K}$ is the controller, Δ_{del} is the perturbation matrix, r is the reference and e is the output.

For the purposes of the algebraic μ-synthesis, the MIMO system with l inputs and l outputs has to be decoupled into l identical SISO plants. The nominal model is defined in terms of transfer functions:

$$\mathbf{P}_{nom}(s) \equiv \begin{bmatrix} P_{11}(s) & \cdots & P_{1l}(s) \\ \vdots & \ddots & \vdots \\ P_{l1}(s) & \cdots & P_{ll}(s) \end{bmatrix} \tag{14}$$

For decoupling the nominal plant $\mathbf{P}_{nom}$ ($\mathbf{P}_{nom}$ invertible) it is satisfactory to have the controller in the form

$$\mathbf{K}(s) = K(s)\mathbf{I}_l \det[\mathbf{P}_{nom}(s)]\frac{1}{P_{xy}(s)}[\mathbf{P}_{nom}(s)]^{-1} \tag{15}$$

where P_{xy} is an element of $adj[\mathbf{P}_{nom}(s)] = \det[\mathbf{P}_{nom}(s)][\mathbf{P}_{nom}(s)]^{-1}$ with the highest degree of numerator $\{adj[\mathbf{P}_{nom}(s)]$ denotes adjugate matrix of $\mathbf{P}_{nom}\}$. The choice of the decoupling matrix prevents the controller from cancelling any poles or zeros from the right half-plane so that internal stability of the nominal feedback loop is held. The MIMO problem is reduced to finding a controller $K(s)$ which is tuned via setting the poles of the nominal feedback loop with the plant

$$\begin{aligned} \mathbf{P}_{dec}(s) &= \frac{1}{P_{xy}(s)}\det[\mathbf{P}_{nom}(s)][\mathbf{P}_{nom}(s)]^{-1}\mathbf{P}_{nom}(s) \\ &= \frac{1}{P_{xy}(s)}\det[\mathbf{P}_{nom}(s)]\mathbf{I}_l \end{aligned} \tag{16}$$

Define

$$P_{dec} \equiv \frac{1}{P_{xy}(s)}\det[\mathbf{P}_{nom}(s)] \tag{17}$$

Transfer function P_{dec} can be approximated by a system P^*_{dec} with lower order than P_{dec}

$$P^*_{dec}(s) = \frac{b(s)}{a(s)} \tag{18}$$

which can be rewritten in terms of its coefficients and transformed to the elements of $\mathbf{R}_{\mathrm{PS}}$

$$P^*_{dec}(s) = \frac{\dfrac{b_0 + b_1 s + \ldots + b_n s^n}{(\alpha_1+s)(\alpha_2+s)\cdot\ldots\cdot(\alpha_n+s)}}{\dfrac{s^n + a_0 + a_1 s + \ldots + a_{n-1}s^{n-1}}{(\alpha_1+s)(\alpha_2+s)\cdot\ldots\cdot(\alpha_n+s)}} = \frac{B}{A},\ A, B \in \mathbf{R}_{\mathrm{PS}} \tag{19}$$

The controller $K = N_K/D_K$ is derived as solution of the Diophantine equation

$$AD_K + BN_K = 1 \tag{20}$$

with A, B, D_K, $N_K \in \mathbf{R}_{\mathrm{PS}}$. Equation (20) is the Bezout identity. All feedback controllers N_K/D_K are given by

$$K = \frac{N_K}{D_K} = \frac{N_{K_0} - AT}{D_{K_0} + BT},\ N_{K_0}, D_{K_0} \in \mathbf{R}_{\mathrm{PS}} \tag{21}$$

where N_{K_0}, $D_{K_0} \in \mathbf{R}_{\mathrm{PS}}$ are particular solutions of (20) and T is an arbitrary element of $\mathbf{R}_{\mathrm{PS}}$.

The controller K satisfying equation (20) guarantees the BIBO (bounded-input bounded-output) stability of the feedback loop in Fig. 4. This is a crucial point for the theorems regarding the structured singular value. If the BIBO stability is held, then the nominal model is internally stable and theorems related to robust stability and performance can be used. The BIBO stability also guarantees stability of $\mathbf{F}_L(\mathbf{G}, \mathbf{K})$ making possible usage of performance weights with integration property implying non-existence of state space solutions using DGKF formulae [see Doyle et al. (1989)] due to zero eigenvalues of appropriate Hamiltonian matrices. This procedure results in zero steady-state error in the feedback loop with the controller obtained as a solution to equation (20) being neither possible in the scope of the standard μ-synthesis using DGKF formulae, nor using LMI approach [see Gahinet and Apkarian (1994)] leading to numerical problems in most of real-world applications.

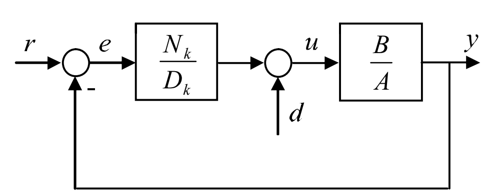

Fig. 4. Nominal feedback loop

The aim of global optimization in the algebraic approach is to design a controller satisfying the condition:

$$\sup_{\substack{\omega \\ K \text{ stabilizing } \mathbf{G}}} \mu_\Delta[\mathbf{F}_L(\mathbf{G},\mathbf{K})(\omega,\alpha_1,\ldots,\alpha_{n+n_1+n_2},t_1,\ldots,t_{n_2})] \le 1,\ \omega \in (-\infty,+\infty) \tag{22}$$

where $n + n_1 + n_2$ is the order of the nominal feedback system, n_1 is the order of particular solution, K_0, t_i are arbitrary parameters in $T = \frac{t_0 + t_1 s + \ldots + t_{n_2} s^{n_2}}{(\alpha_{n_1+1} + s)\cdot\ldots\cdot(\alpha_{n_1+n_2} + s)}$ and $\mu_{\mathbf{\Delta}}$ denotes the structured singular value of LFT on generalized plant **G** and controller **K** with **Δ** defined in (12).

Tuning parameters are positive and constrained to the real axis since parameters of the transfer function have to be real and due to the fact that non-real poles cause oscillations of the nominal feedback loop.

A crucial problem of the cost function in (22) is the fact that many local extremes are present. Hence, local optimization does not yield a suitable or even stabilizing solution. This can be overcome via evolutionary computation solving the task very efficiently.

5. EXAMPLE - PROBLEM FORMULATION & SOLUTION

The problem to solve is 2nd order system with 1st order astatism and uncertain time delay:

$$\mathbf{P} \equiv \left\{ \frac{b_0}{a_2 s^2 + a_1 s} e^{-\tau s} \right\} \tag{23}$$

$a_1 \in [1.8, 2.2]$, $a_2 = 1$,
$b_0 \in [1.8, 2.2]$, $\tau \in [0, 4]$

The control objective is to find a controller for which the robust stability and performance is held for every plant from the set **P**. The weights follow from (6) and (7):

$$W_{a1} = 0.2\,,\; W_{b0} = 0.2 \tag{24}$$

The time delay is treated by multiplicative uncertainty (see Fig. 1)

$$\{P(1 + W_{del}\delta_{del}) : |\delta_{del}| \le 1, \delta_{del} \in \mathbf{C}\} \tag{25}$$

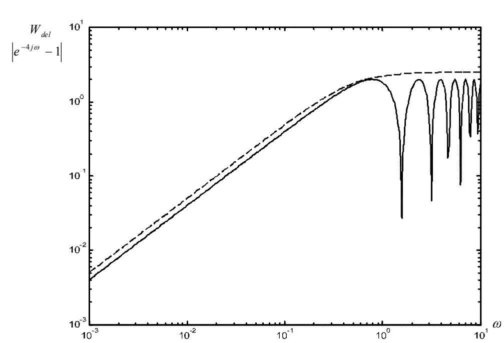

Fig. 5. Bode plot of W_{del} and $\left|e^{-4j\omega} - 1\right|$

Let the nominal plant be

$$P(s) = \frac{b}{a} = \frac{2}{s^2 + 2s} \tag{26}$$

then for the weighting function W_{del} the following inequality must be held $\mathbf{P}'$ being the set **P** omitting the parametric uncertainties

$$\left|\frac{P'(j\omega)}{P(j\omega)} - 1\right| < |W_{del}(j\omega)|\,,\; \forall\omega \in \overline{\mathbf{R}}_+ \;\; \forall P' \in \mathbf{P} \tag{27}$$

which is equivalent with

$$\left|e^{-\tau j\omega} - 1\right| < |W_{del}(j\omega)|\,,\; \forall\omega \in \overline{\mathbf{R}}_+\,,\; \tau \in [0; T_d] \tag{28}$$

The weight W_{del} is defined as an envelope curve of $\left|e^{-\tau j\omega} - 1\right|$. For $\tau = 4$, W_{del} can have the Bode plot depicted in Fig. 5:

$$W_{del}(s) = 2.5\frac{2s}{2s + 1} \tag{29}$$

The performance condition is of the form:

$$\|W_1 S\|_\infty < 1 \tag{30}$$

where S is the sensitivity function and weight W_1 (see Fig. 2) is defined for the algebraic approach and D-K iteration as follows:

$$W_1^A(s) = \frac{0.004}{10s^3 + 100s^2 + s + 10^{-5}} \cdot 50 \tag{31}$$

$$W_1^{D-K}(s) = \frac{0.004}{10s^3 + 100s^2 + s + 10^{-5}} \cdot 50 \tag{32}$$

By the optimization of the poles α_i via the Differential Migration and subsequent tuning by the Nelder-Mead simplex method, resulting poles were obtained:

$$\alpha_1 = 0.065,\; \alpha_2 = 0.063,\; \alpha_3 = 2.021,\; \alpha_4 = 62.338 \tag{33}$$

yielding the controller

$$K_A(s) = \frac{4.618s^2 + 8.182s + 0.2571}{s^2 + 62.49s} \tag{34}$$

The controller obtained from the D-K iteration was approximated by the 3rd order transfer function:

$$K_{D-K}(s) = \frac{0.332s^3 + 0.968s^2 + 0.0442s + 0.0003}{s^3 + 8.926s^2 + 0.0692s + 6.998\cdot 10^{-7}} \tag{35}$$

The μ-plot in Fig. 8 shows that both controllers have the supremum of μ below one and the robust stability and performance condition is satisfied.

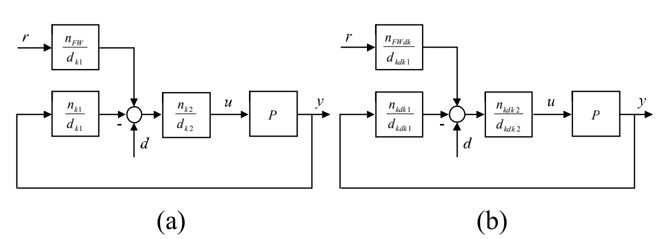

Fig. 6. 2DOF feedback loop

The controllers for 2DOF feedback loop (Fig. 6a, 6b - algebraic approach and D-K iteration, respectively) have the compensator (n_{k2}, d_{k2}, n_{kdk2}, d_{kdk2}) defined as fraction of the factors corresponding with most stable zero and least stable pole of K_A and K_{D-K} and feedback (n_{k1}, d_{k1}, n_{kdk1}, d_{kdk1}) and feed-forward part (n_{FW}, d_{k1}, n_{FW}, d_{kdk1}) defined by the fraction of the factors corresponding with remaining zeros and poles of K_A and $K_{D\text{-}K}$ with $n_{FW} = n_{k1}^0$ and $n_{FWdk} = n_{kdk1}^0$ (n_{k1}^0, n_{kdk1}^0 being the coefficients of n_{k1} and n_{kdk1} of zero exponent of s):

$$\frac{n_{k1}}{d_{k1}} = \frac{4.618s + 0.1478}{s^2 + 62.49s},\ \frac{n_{FW}}{d_{k1}} = \frac{0.1478}{s^2 + 62.49s},\ \frac{n_{k2}}{d_{k2}} = \frac{s + 1.74}{s} \quad (36)$$

$$\frac{n_{kdk1}}{d_{kdk1}} = \frac{0.3316s^2 + 0.0154s + 9.13\cdot 10^{-5}}{s^2 + 8.926s + 0.0691},\ \frac{n_{FWdk}}{d_{kdk1}} = \frac{9.13\cdot 10^{-5}}{s^2 + 8.926s + 0.0691},\ \frac{n_{kdk2}}{d_{kdk2}} = \frac{s + 2.87}{s + 1\cdot 10^{-5}} \quad (37)$$

The periodicity is defined via sinusoids changing the uncertain parameters in the intervals defined by (23):

$$b_0 = \overline{b}_0[1 + \lambda_{b_0} \sin(\omega_{b_0} t)] \quad (38)$$

where $\overline{b}_0 = 2$, $\lambda_{b_0} = 0.2$ and $\omega_{b_0} = 1$. The step response for the periodic change (38) is in Fig. 7.

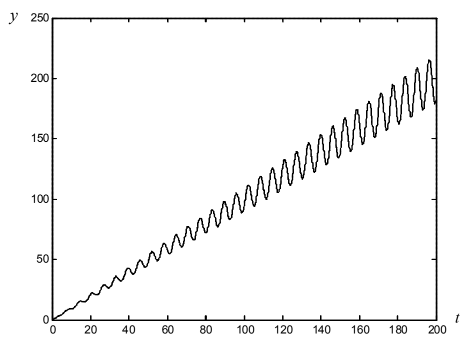

Fig. 7. Step response for periodic changes of parameters

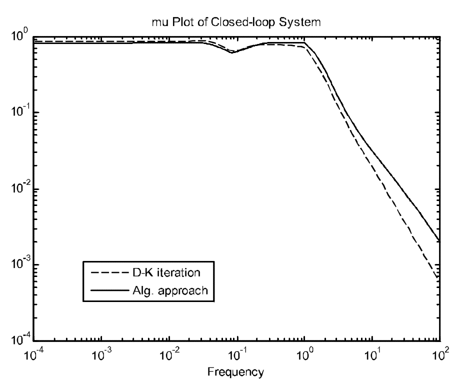

Fig. 8. μ-plots for the controllers obtained by the *D-K* iteration and algebraic approach

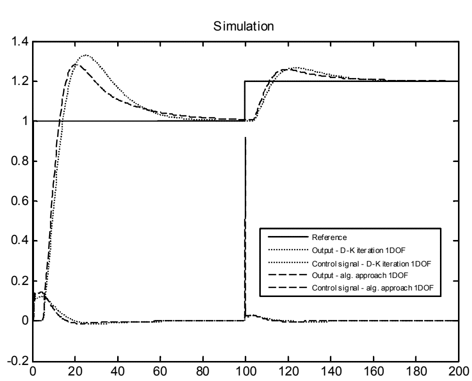

Fig. 9. Simulations for simple feedback loop without periodic changes – algebraic approach and *D-K* iteration

Simulations in Fig. 9 and 10 show that *D-K* iteration yields non-zero steady-state error in contrast to the algebraic approach having no steady state error and faster set point tracking than the *D-K* iteration controller. Simulations for periodic changes (38) in Fig. 11 and 12 prove that the 1DOF and 2DOF feedback loops are stable for both the algebraic approach and *D-K* iteration. In all simulations full time delay is connected, i.e. τ = 4 s.

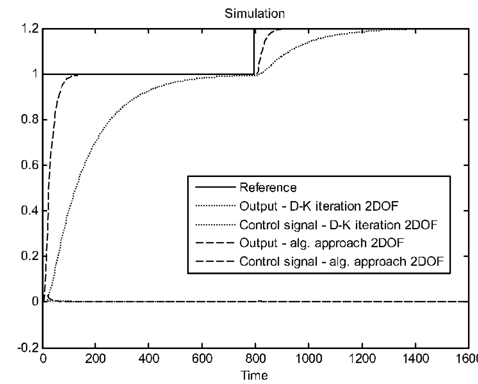

Fig. 10. Simulations for 2DOF feedback loop without periodic changes – algebraic approach and *D-K* iteration

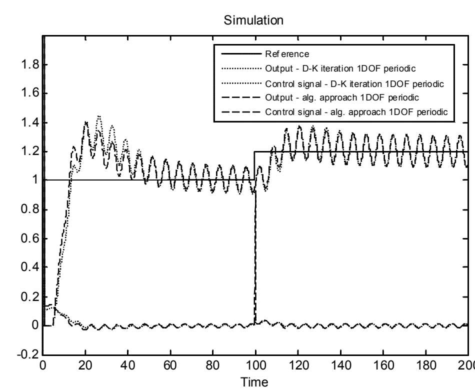

Fig. 11. Simulations for simple feedback loop with periodic changes – algebraic approach and *D-K* iteration

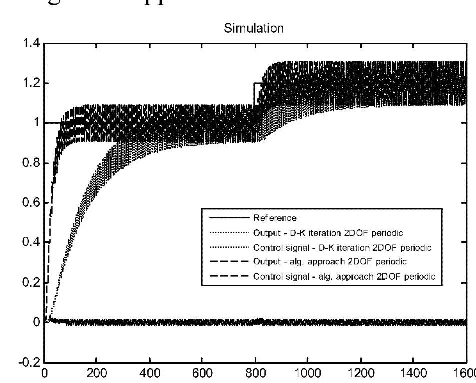

Fig. 12. Simulations for 2DOF feedback loop with periodic changes – algebraic approach and *D-K* iteration

6. USER INTERFACE

The main window of the Matlab toolbox consists of three sections (see Fig. 13):

- System Definition
- Controller Design
- Simulation and Verification

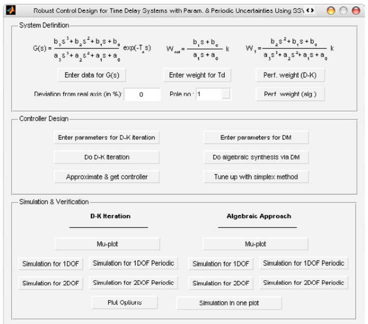

Fig. 13. The main window

6.1 System Definition

System definition has the button for displaying the dialog for entering parameters of the control plant where the parameters of transfer function, the maximum value of time delay and parameters for periodic changes can be entered (Fig. 14).

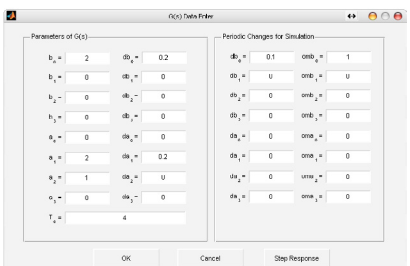

Fig. 14. Dialog for entering parameters of the controlled plant

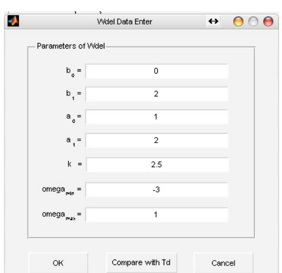

Fig. 15. Dialog for entering the parameters of the weight W_{del}

Another button displays the dialog for entering the parameters of the weight W_{del} treating uncertain time delay (Fig. 15) with button showing the Bode plot of the weight W_{del} compared to the left side of (28).

In the last part of system definition, buttons showing dialogs for entering parameters of the performance weight W_1 are placed. There are separate weights for the *D-K* iteration and algebraic approach. Each dialog has a button for showing the Bode plot of the particular weight.

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