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SOLUTION OF INVARIANCY OF DOUBLE-VARIABLE CONTROL CIRCUIT (PART 1)

Balátě, J.; Krupková, M. & Navrátil, P.

Abstract: *This paper deals with solution of invariancy. It describes two methods of its using a school-example of double-variable control circuit. The first method solves the invariancy by using so called adapted biding controllers $R^u(s)$ and the second method solves the invariancy by means of correction members KC(s). The paper deals with difficulties of the first method and with benefits of the second method. Results of both methods are supported by verifying simulation. The verifying simulations were carried out at both methods on two control schemes of a double-variable control circuit.*

Key words: *invariancy, control circuit, correction member, binding controllers*

1. INVARIANCY OF CONTROL CIRCUIT

The aim of invariancy solution is to eliminate influence of a failure affecting the control circuit; generally to achieve that the control circuit eliminates the influence of failure affections and therefore that these failures do not affect control process of a controlled system (Åström & Hägglund, 1994). For objective information we use double-variable control circuit (see Fig. 1).

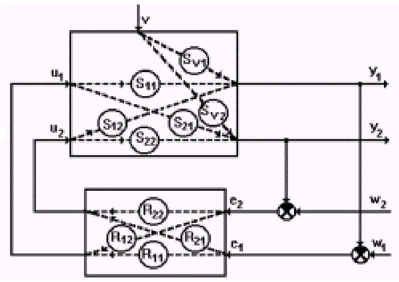

Fig. 1. Double-variable control circuit

Transfers of intrinsic (on the main diagonal) and extrinsic (out of the main diagonal) action variables are given for this double-variable control circuit (1)

$$\mathbf{G_S}(s) = \begin{pmatrix} \dfrac{2}{s^2+5s+6} & \dfrac{0.1}{2s+1} \\ \dfrac{0.1}{2s+1} & \dfrac{2}{s^2+5s+6} \end{pmatrix} \quad (1)$$

and transfers of main controllers (on the main diagonal) (2), which have been calculated by the method of dynamics inversion (Vítečková, 2000; Wagnerová & Minář, 2000).

$$\mathbf{G_R}(s) = \begin{pmatrix} \dfrac{1.16s+3.49}{s^2+1.9s} & R_{12}(s) \\ R_{21}(s) & \dfrac{1.16s+3.49}{s^2+1.9s} \end{pmatrix} \quad (2)$$

$R_{12}(s)$ and $R_{21}(s)$ are binding controllers (controllers out of the main diagonal) complying with the condition of autonomy.

Vector of a failure variable is given by

$$\mathbf{v}(s) = \begin{pmatrix} S_{V1}(s) \\ S_{V2}(s) \end{pmatrix} = \begin{pmatrix} \dfrac{0.1}{s+1} \\ \dfrac{0.1}{s^2+2s+1} \end{pmatrix} \quad (3)$$

This model of double-variable control circuit serves as a school-example with the given matrix of action variable $G_S(s)$ (1) and the failure **v**(s) (3).

2. METHODS OF INVARIANCY SOLUTION INCLUDING VERIFYING THE RESULTS OF SIMULATIONS

Two approaches to invariancy solution of double-variable control circuit were used.

2.1 Using so called adapted binding controllers $R^u_{12}(s)$ and $R^u_{21}(s)$

Transfers of adapted binding controllers $R^u_{12}(s)$ (8) and $R^u_{21}(s)$ (10) are calculated on the base of invariancy requirement (7),(9).

For determining the condition of circuit invariancy with regard to a failure **v** we adapt the system of equations of the double-variable control circuit

$$\begin{aligned} Y_1 &= (W_1 - Y_1)(R_{11}S_{11} + R_{21}S_{12}) + (W_2 - Y_2)(R_{12}S_{11} + R_{22}S_{12}) + VS_{V1} \\ Y_2 &= (W_1 - Y_1)(R_{11}S_{21} + R_{21}S_{22}) + (W_2 - Y_2)(R_{12}S_{21} + R_{22}S_{22}) + VS_{V2} \end{aligned} \quad (4)$$

in such way that we carry out a separation of variables i.e. that controlled variables are on the left and failure variable and required values of controlled variables are on the right sides of equations.

If we designate various factors in equation (4)

$$\begin{aligned} R_{11}S_{11} + R_{21}S_{12} &= A \quad & R_{12}S_{11} + R_{22}S_{12} &= B \\ R_{11}S_{21} + R_{21}S_{22} &= C \quad & R_{12}S_{21} + R_{22}S_{22} &= D \end{aligned} \quad (5)$$

then the solution of this system of equations is

$$\begin{aligned} Y_1[(1+A)(1+D) - BC] &= V[S_{V1}(1+D) - S_{V2}B] + W_1[A(1+D) - BC] + W_2B \\ Y_2[(1+D)(1+A) - BC] &= V[S_{V2}(1+A) - S_{V1}C] + W_1C + W_2[D(1+A) - BC] \end{aligned} \quad (6)$$

The condition of circuit invariancy with regard to the failure **v**(s) is obvious from these equations i.e. the circuit is invariant provided the factors in the bracket at the failure variable **v**(s) in the equation for the respective variable y_i equal to zero. Then it holds:

- for independence of y_1 with regard to the failure **v**

$$S_{V1}(1+D) - S_{V2}B = 0$$

$$\frac{S_{V1}}{S_{V2}} = \frac{R^u_{12}S_{11} + R_{22}S_{12}}{1 + R^u_{12}S_{21} + R_{22}S_{22}} \quad (7)$$

From the equation (7) it is possible to determine the transfer of the adapted binding controller $R^u_{12}(s)$:

$$\Rightarrow \quad R^u_{12}(s) = -\frac{4s^6 + 20.6s^5 + 52.7s^4 + 80.5s^3 + 69.7s^2 + 32.2s + 4.9}{-0.1s^5 - 0.79s^4 + 1.76s^3 + 6.91s^2 + 2.66s} \quad (8)$$

- for independence of y_2 with regard to the failure **v**

$$S_{V2}(1+A) - S_{V1}C = 0$$

$$\frac{S_{V2}}{S_{V1}} = \frac{R_{11}S_{21} + R^u_{21}S_{22}}{1 + R_{11}S_{11} + R^u_{21}S_{12}} \quad (9)$$

From the equation (9) it is possible to determine the transfer of the adapted binding controller $R^u_{21}(s)$:

$$\Rightarrow \quad R^u_{21}(s) = -\frac{4s^5 + 18.5s^4 + 36.9s^3 + 39.6s^2 + 23.2s + 4.9}{3.9s^4 + 12.91s^3 + 11.85s^2 + 2.66s} \quad (10)$$

By fulfilling one of both conditions (7) and (9) double-variable control circuit with both controlled variables y_1 and y_2 comes into being. One of these controlled variable i.e. y_1 or y_2 (but only one of them) is statically (in stabilized state) and also dynamically (during transition process) independent on effect of failure variable **v**. The circuit is therefore absolutely invariant only for one selected controlled variable (Balátě, 1996).

At the calculation according to the theory of adapted binding controllers $R^u_{12}(s)$ (8) and $R^u_{21}(s)$ (10) we have met a difficulty:

1. We remind that by described adaptation for double-parameter control circuit ***it is possible to ensure the invariancy of only one (selected) controlled variable***, i.e. the influence only to one element of the failure vector **v**(s) is eliminated.
2. ***Fulfilling of the physical feasibility condition*** is a further difficulty. It was not possible to work with the selected adapted binding controller $R^u_{21}(s)$ (10), because as it is obvious the condition of physical feasibility was not fulfilled, because the digit position of the polynomial grade of denominator was lower than of numerator. Due to this reason it was necessary to extend the polynomial grade of denominator by a member with first grade inertia 1/(s+1) and thus at least weak condition of physical feasibility was achieved. After extending by the member with first grade inertia, the adapted binding controller has this form

$$R^u_{21}(s) = -\frac{4s^5 + 18.5s^4 + 36.9s^3 + 39.6s^2 + 23.2s + 4.9}{3.9s^5 + 16.81s^4 + 24.76s^3 + 14.51s^2 + 2.66s} \quad (11)$$

Verifying function of double-variable control circuit invariancy (method 1)

The second binding controller $R_{12}(s)$ was gained on the base of the autonomy condition

$$R_{12}S_{11} + R_{22}S_{12} = 0$$

$$R_{12}(s) = -\frac{S_{12}(s)}{S_{11}(s)} R_{22}(s) = -\frac{0.058s^3 + 0.465s^2 + 1.221s + 1.047}{2s^3 + 4.8s^2 + 1.9s} \quad (12)$$

Verifying simulation of the invariancy function was carried out on the connection of double-variable control circuit (in the environment MATLAB-Simulink) according to the Fig. 2.

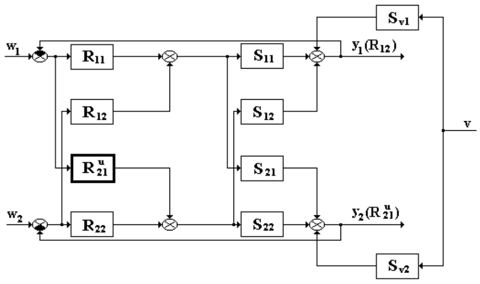

Fig. 2. Connection of double-variable control circuit with adapted binding controller $R^u_{21}(s)$

For an analysis and better comparison of functional operation of the control circuit connected according to Fig. 2, yet another control circuit was used, which differed from the preceding in such way that binding controller $R_{21}(s)$ was used in place of adapted binding controller $R^u_{21}(s)$ which was determined from the condition of autonomy and is identical with binding controller $R_{12}(s)$ (12) due to symmetrical form of transfer matrix of action variables (1). Connection of this control circuit is in the Fig. 3.

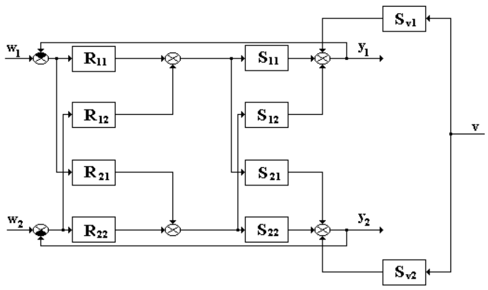

Fig. 3. Connection of double-variable control circuit

By means of this verifying simulation on these connections we tried to find out how the adapted binding controller influences invariancy and how autonomy is displayed when using the adapted binding controller. Influence of the adapted binding controller on invariancy is shown in the Fig. 4 where required values w_1 and w_2 of the controlled variable are set to the value = 0 and the failure variable **v** is set to the value = 1.

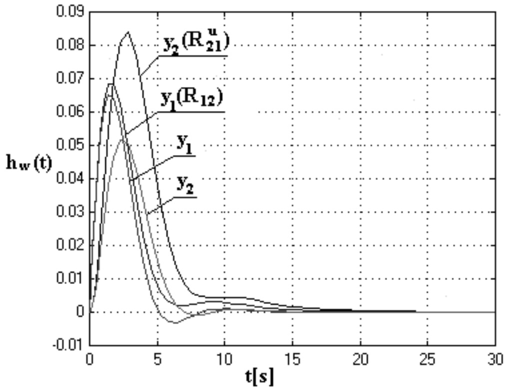

Fig. 4. Influence of the adapted binding controller on invariancy ($w_1 = w_2 = 0$ and **v** = 1)

It is obvious from Fig. 4 that influence of the failure variable was not eliminated in case of $y_1(R_{12})$ because it is nearly identical with the course of controlled variable y_1; that shows the difficulty described in the point 1. Effect of the failure variable was not eliminated even in case of $y_2(R^u_{21})$.

Influence of adapted binding controller on autonomy is shown in the Fig. 5 where on the contrary the required values w_1 and w_2 are set to the required value = 1 and the failure variable **v** is set to the value = 0.

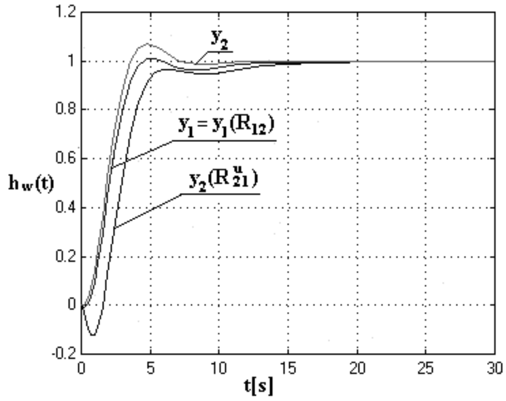

Fig. 5. Influence of the adapted binding controller on autonomy ($w_1 = w_2 = 1$ and **v** = 0)

It is obvious from Fig. 5 that due to adapted binding controller $R^u_{21}(s)$ there occurred a disorder of autonomy, because the courses of output (controlled) variable $y_2(R^u_{21})$ and y_2 are not identical (they do not cover one another) as it is in case of y_1 and $y_1(R_{12})$, which cover one another and therefore there autonomy was not infringed.

Fig. 6 displays both cases, the adapted binding controller influence on invariancy, as well as on autonomy. There the required values and also the failure variable are set to the value = 1.

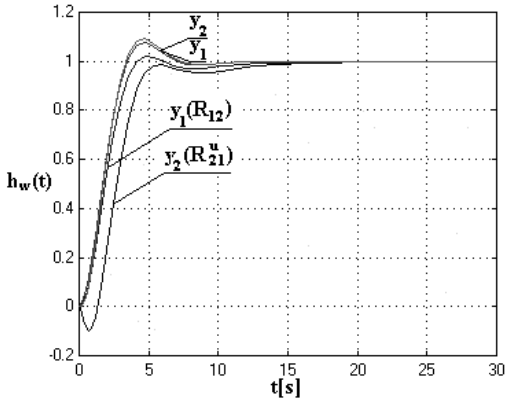

Fig. 6. Influence of the adapted binding controller on autonomy and on invariancy ($w_1 = w_2 = 1$ and **v** = 1)

It is obvious from Fig. 6 that no suppression of failure variable occurred and that substantial disorder of autonomy occurred particularly at $y_2(R^u_{21})$.

SOLUTION OF INVARIANCY OF DOUBLE-VARIABLE CONTROL CIRCUIT (PART 2)

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Abstract: *This paper deals with solution of invariancy. It describes two methods of its using a school-example of double-variable control circuit. The first method solves the invariancy by using so called adapted biding controllers $R^u(s)$ and the second method solves the invariancy by means of correction members KC(s). The paper deals with difficulties of the first method and with benefits of the second method. Results of both methods are supported by verifying simulation. The verifying simulations were carried out at both methods on two control schemes of a double-variable control circuit.*

Key words: *invariancy, control circuit, correction member, binding controllers*

2.2 Using correction members

The preceding approach to invariancy solution of double-variable control circuit has shown difficulties of using adapted binding controllers $R^u(s)$. These difficulties have guided to an idea to look for another solution. An idea came into being to use ***an analogy of single-variable branched control circuit with assigning of a failure variable*** (see Fig. 7) for the solution of invariancy (Balátě, 1996;Balátě, 2002).

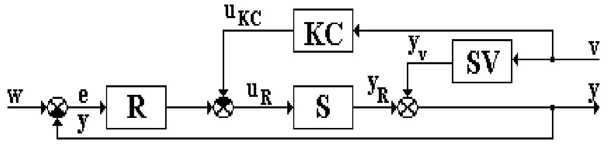

Fig. 7. Block scheme of single-variable control circuit with assigning of failure variable

For the single-variable branched control circuit with assigning of the failure variable (Fig. 7), the condition for calculation of ***correction member transfer function*** must be valid (13)

$$G_v(s) = \frac{Y(s)}{V(s)} = \frac{G_{SV}(s) - G_S(s)G_{KC}(s)}{1 + G_S(s)G_R(s)} = 0 \qquad (13)$$

in order to eliminate influence of the failure (Balátě, 1996)

$$G_{SV}(s) = G_S(s)G_{KC}(s) \qquad (14)$$

where $G_{KC}(s)$ is the transfer function of correction member.

For comparability of results there was used for solution the double-variable control circuit with given matrix of action variables $G_S(s)$ (1), failure vector **v**(s) (3) and calculated matrix controller $G_R(s)$ (2).

On the base of the condition (14) for a single-variable branched control circuit with assigning of a failure variable there were designed correction members

$$KC_1(s) = \frac{S_{V1}(s)}{S_{11}(s)} = \frac{0.05s^2 + 0.25s + 0.3}{s+1} \qquad (15)$$

and

$$KC_2(s) = \frac{S_{V2}(s)}{S_{22}(s)} = \frac{0.05s^2 + 0.25s + 0.3}{s^2 + 2s + 1} \qquad (16)$$

The result conducts to ***more simple calculation of correction members*** and at it the resulting form of correction members is also more simpler. At this method there ***can not occur originating of negative coefficients*** in the denominator KC(s).

It is necessary to pay attention to fulfilling the condition of these members physical feasibility. Also in this example this problem occurred namely in case of the correction member $KC_1(s)$. It was necessary to extend this correction member at least by a member with first grade inertia. Thus the weak condition of physical feasibility was achieved

$$KC_1(s) = \frac{0.05s^2 + 0.25s + 0.3}{s+1} \cdot \frac{1}{s+1} = \frac{0.05s^2 + 0.25s + 0.3}{s^2 + 2s + 1} \qquad (17)$$

Advantage of this method is that there does not occur the influence of autonomy by adapted binding controller $R^u(s)$ (what we have observed at the first described method).

At this method correction member $KC_1(s)$, $KC_2(s)$ are assigned into the control circuit. Binding controllers $R_{12}(s)$, $R_{21}(s)$ are designed by means of the condition of autonomy and are of the type (12). This condition is displayed in the Fig. 8.

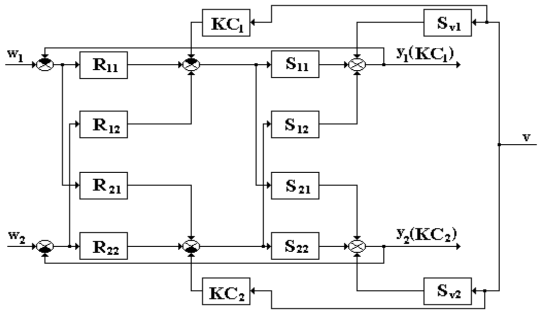

Fig. 8. Connection of double-variable control circuit with correction members $KC_1(s)$ and $KC_2(s)$

Verifying of the function of double-variable control circuit invariancy (method 2)

Verifying was carried out on the connection of double-variable control circuit according to the Fig. 8 in the environment MATLAB-Simulink. For an analysis and better comparison there was used the connection of double-variable control circuit without correction member $KC_1(s)$, $KC_2(s)$ (see Fig. 9). Again we have been finding out how the correction members influence invariancy and how autonomy.

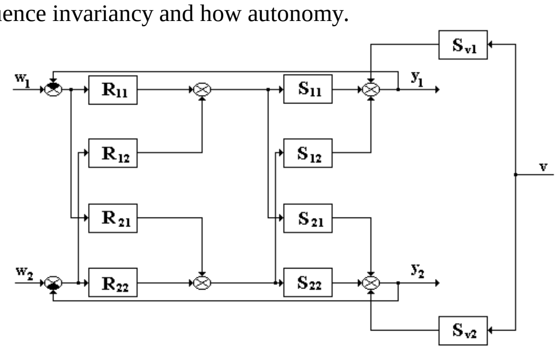

Fig. 9. Connection of double-variable control circuit

Influence of correction members on invariancy is displayed in the Fig. 10 where the required values w_1 and w_2 are set to zero and the failure variable **v** is set to one.

It is obvious from Fig. 10 that the failure variable is partially eliminated due to the influence of KC(s). This elimination is substantial in case of the course of controlled variable $y_2(KC_2)$.

This elimination of the failure variable is worse in case of $y_1(KC_1)$ and this is caused by infringing the condition (14), because $KC_1(s)$ is extended by the member with first grade due to the requirement of physical feasibility.

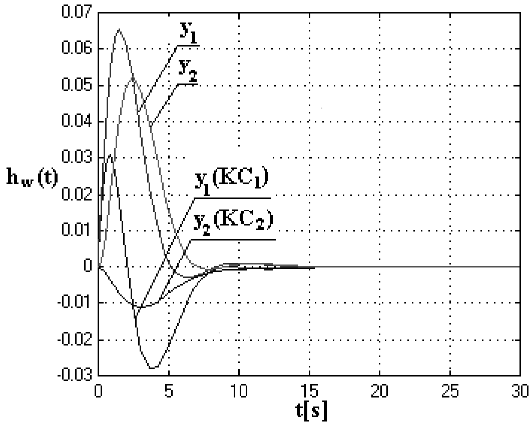

Fig. 10. Influence of correction members KC(s) on invariancy ($w_1 = w_2 = 0$ and **v** = 1)

Influence of KC(s) on autonomy is displayed in the Fig. 11 where the required values w_1, w_2 are set to the required value 1 and the failure variable **v** is set to 0.

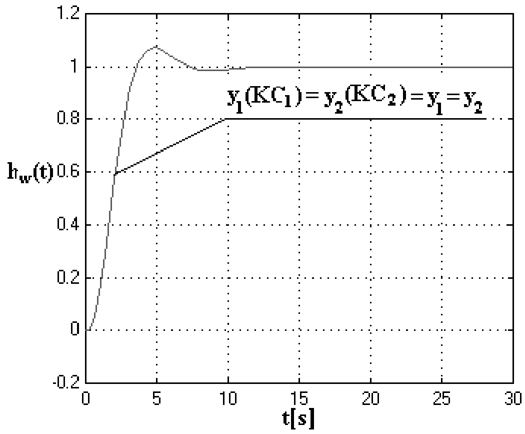

Fig. 11. Influence correction members KC(s) on autonomy ($w_1 = w_2 = 1$ and **v** = 0)

It is obvious from Fig. 11 that there the condition of autonomy is fulfilled, because the courses of controlled (output) variables cover one another (they are identical).

Fig. 12 records the influence of KC(s) on invariancy and on autonomy. The required values w_1, w_2 and also the failure variable **v** are there set to the value 1.

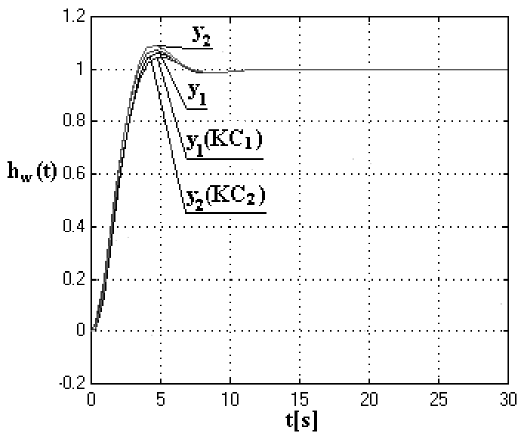

Fig. 12. Influence of the KC(s) on autonomy and on invariancy ($w_1 = w_2 = 1$ and **v** = 1)

It is obvious from Fig. 12 that a failure is partially eliminated in double-variable control circuit and the control circuit is nearly autonomous (autonomy is infringed by the effect of failure variable).

3. PRESENT SITUATION

At present we are involved in solution of invariancy at triple-variable control circuit, which is characterized by a real two-off-take steam turbine and mathematic model of controlled system corresponds to the above mentioned limits i.e. that failures are measurable

- ΔM_G change of load with influence on angular speed $\Delta\omega$ of a turbo-set and
- change of weight-flow of off-take steam $\Delta\dot{m}_{E1}$, $\Delta\dot{m}_{E2}$ on steam pressure in corresponding off-takes $\Delta\dot{p}_{E1}$, $\Delta\dot{p}_{E2}$

4. EVALUATION AND CONCLUSION

It results from the above outcomes that the first method i.e. using adapted binding controllers $R^u(s)$ does not bring expected results for solution of invariancy of double-variable control circuit. It is more advantageous to use the second method of solution where correction members KC(s) are used. This method eliminates better the failure variable **v**, correction members do not influence autonomy and there is no danger of originating negative coefficients in denominators of these KC(s) and thus negative coefficients can not originate in the characteristic equation of closed control circuit.

The method of invariancy solution appears as being well useful also for multi-variable control circuits in case when separate failures are important always for one of vector elements of controlled variables.

5. REFERENCES

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Authors: Prof. Dipl. Ing. Jaroslav BALÁTĚ, DrSc., Dipl. Ing. Markéta KRUPKOVÁ, Dipl. Ing. Pavel NAVRÁTIL; Tomas Bata University in Zlin, Faculty of Technology, Institute of Information Technologies, Mostni 5139, 760 01 Zlin, Czech Republic, URL: http://www.ft.utb.cz/czech/iit, Phone: +420 576033210, Fax: +420 576033333, E-mail: balate@ft.utb.cz, m_krupkova@ft.utb.cz, p_navratil@ft.utb.cz