Active damping of controlled mechanic systems

ZDENĚK ÚŘEDNÍČEK
Department of Automation and Control
Tomas Bata University in Zlin
Jižní Svahy, Nad Stráněmi 4511, 760 05, Zlin
CZECH REPUBLIC
urednicek@fai.utb.cz http://www.utb.cz/fai-en/structure/zdenek-urednicek

Abstract: - Paper describes active damping possibility of mechanical system with two not ideally stiffly connected masses. This type of problem can by often see at speed or position servo systems with electromechanical actuators – electrical engines. There controlled values are usually measured directly on actuator and not on loading mass. This is a reason why the control precision depends on elimination of load mass to the actuator torque influence in control system. At many motion control tasks, the problem of oscillations existence in multidimensional system with limited motion control and imperfect or complicated state quantities measurement possibility exists. Paper also describes active damping simple possibility of these type systems and by two-dimensional system physical model shows active damping possibility also with indirect state quantities measurement option

Key-Words: - Motion control, active damping, physical model, multiport model, observer

1 Introduction

Frequent problem at mechanical system motion precision with speed or position control by actuator formed by electric or hydraulic engines is consequence of fact that controlled value sensing is made commonly on motor, which causes that mechanical part, which is our interest subject, is controlled indirectly.

This problem is presented significantly in moment when between actuator mass (electric machine) and load mass isn't ideally rigid connection. It's in some detail common problem of any transmissivity arrangement containing e.g. the play, but especially significantly such system behavior demonstrates in case of e.g. harmonics gearbox utilization.

This article deals first of all with one simple solution of mentioned problem and forms some simplified starting point for next part, which solves mentioned problem by means of full active damping with incomplete observer.

In second part the paper goal is to introduce some pieces of knowledge relevant to active damping of mechanical systems with more degree of freedom with limited action interventions' possibilities and limited or complicated quantities measurement possibility. This problem often occurs at different mechanical systems motion control types serving as optical (surveillance) or other systems porter, which depend on effector systems positional state accuracy, and when actuators functions in some generalized coordinates only.

As such system example can be cameras porter, laser scanning system porter created from no ideally stiff bodies, weapons porter system with uncontrolled projectiles, but also manipulator with no ideally stiff arms for exact assembly application, not to mention, for invasive medicine application.

2 First Problem Formulation

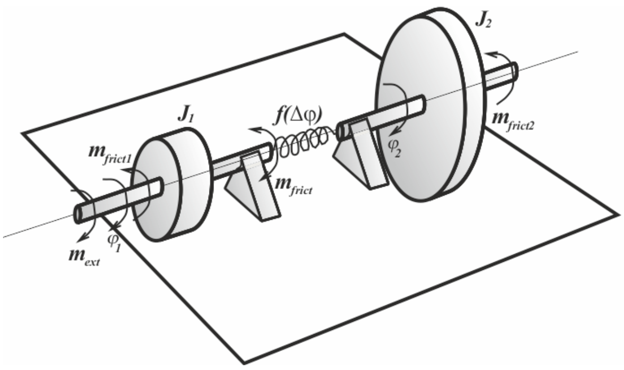

Fig.1 Two rotating masses system with no rigid mechanic interconnection by linear spring.

If we suppose linear (ideal) torsion spring between both masses with stiffness **k** and with linear friction in its material, then motional equations system describing this arrangement is:

$$\begin{aligned} J_1 \cdot \ddot{\varphi}_1 + (b_1 + b) \cdot \dot{\varphi}_1 - b \cdot \dot{\varphi}_2 + k \cdot \varphi_1 - k \cdot \varphi_2 &= m_{ext} \\ J_2 \cdot \ddot{\varphi}_2 + (b_2 + b) \cdot \dot{\varphi}_2 - b \cdot \dot{\varphi}_1 + k \cdot \varphi_2 - k \cdot \varphi_1 &= 0 \end{aligned} \quad (1)$$

where

b_1, b_2 are viscous friction coefficients in mass $\mathbf{m_1}$ and $\mathbf{m_2}$ bearings,

b is viscous friction coefficient in spring material.

If we think over linear cascade regulator according to Fig. 2, (position P - regulator and speed PI-regulator), then it can be derive for Laplace pictures vector

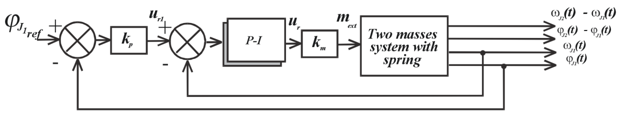

Fig. 2 Masse $\mathbf{J_1}$ cascade position control

$$\begin{bmatrix} \varphi_1(s) \\ \varphi_1(s) \end{bmatrix} = \frac{\varphi_{1ref}(s)}{\det A} \cdot \begin{bmatrix} \left[J_2 \cdot s^2 + (b+b_2)\cdot s + k\right] \cdot \left(k_m \cdot k_{p\omega} \cdot k_p \cdot s + \frac{k_m}{\tau} \cdot k_p\right) \\ (b \cdot s + k) \cdot \left(k_m \cdot k_{p\omega} \cdot k_p \cdot s + \frac{k_m}{\tau} \cdot k_p\right) \end{bmatrix} \quad (2)$$

where

$$\det A = \begin{bmatrix} J_1 \cdot s^3 + \left(k_m \cdot k_{p\omega} + b + b_1\right) \cdot s^2 + \\ + \left(k_m \cdot k_{p\omega} \cdot k_p + \frac{k_m}{\tau} + k\right) \cdot s + \frac{k_m}{\tau} \cdot k_p \end{bmatrix} \cdot$$

$$\cdot \left[J_2 \cdot s^2 + (b+b_2) \cdot s + k\right] - s^2 \cdot (b \cdot s + k)^2$$

For concrete parameters

$J_1 = 10^{-3}\ kgm^2$; $J_2 = 3 \cdot 10^{-3}\ kgm^2$; $b_1 = 0.1\ Nm \cdot s/rad$

$b_2 = 0.2\ Nm \cdot s/rad$; $k = 1500 Nm/rad$; $k_m = 15 Nm/1V$;

$b = 3\ Nm \cdot s/rad$; $k_p = 5$; $k_{pw} = 4$; $\tau = 10^{-2} s$, we obtain

$$\varphi_1(s) = 3.5810^8 \cdot \frac{(s+20) \cdot [s - (-4.97.87 - i \cdot 808..9)] \cdot [s - (-4.97.87 + i \cdot 808..9)]}{(s+20) \cdot \left(s + 1.194378 \cdot 10^5\right) \cdot (s + 3004.81) \cdot [s - (-493.27 + i \cdot 810.49)] \cdot [s - (-493.27 - i \cdot 810.49)]}$$

$$\varphi_2(s) = 3.565745.10^{11} \cdot \frac{(s+20) \cdot (s + 906.0506)}{(s+20) \cdot (s + 3004.81) \cdot \left(s + 1.194378 \cdot 10^5\right) \cdot [s - (-493.27 + i \cdot 810.49)] \cdot [s - (-493.27 - i \cdot 810.49)]}$$

So, it reads:

$$\varphi_1(s) \approx \frac{3.5810^8}{\left(s + 1.194378 \cdot 10^5\right) \cdot (s + 3004.81)} = \frac{3.5810^8}{3.5810^8 \cdot \left(1 + 3.328.10^{-4} \cdot s\right) \cdot \left(1 + 8.373.10^{-6} \cdot s\right)} =$$

$$= \frac{1}{\left(1 + 3.328.10^{-4} \cdot s\right) \cdot \left(1 + 8.373.10^{-6} \cdot s\right)}$$

$$\varphi_2(s) \approx 3..65745.10^{11} \frac{906.0506 \cdot \left(1 + 1.104.10^{-2} \cdot s\right)}{3.5810^8 \cdot \left(1 + 3.328.10^{-4} \cdot s\right) \cdot \left(1 + 8.373.10^{-6} \cdot s\right) \cdot [s - (-493.27 + i \cdot 810.49)] \cdot [s - (-493.27 - i \cdot 810.49)]} =$$

$$= 6.0347.10^5 \cdot \frac{\cdot \left(1 + 1.104.10^{-2} \cdot s\right)}{\left(1 + 3.328.10^{-4} \cdot s\right) \cdot \left(1 + 8.373.10^{-6} \cdot s\right) \cdot [s - (-493.27 + i \cdot 810.49)] \cdot [s - (-493.27 - i \cdot 810.49)]}$$

Consequently, for concrete parameters we obtain the transient and frequency characteristic of $\varphi_1(s)$ - Fig. 3 and Fig. 4.

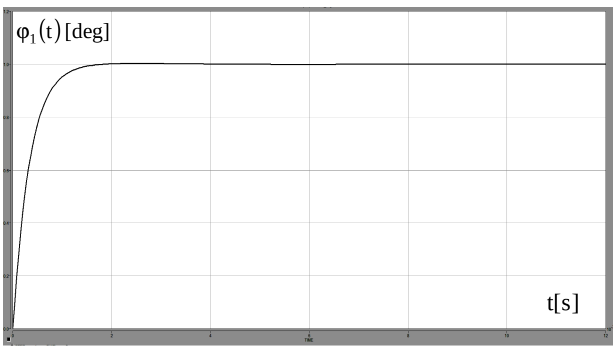

Fig. 3 $\varphi_1(s)$ transient characteristic

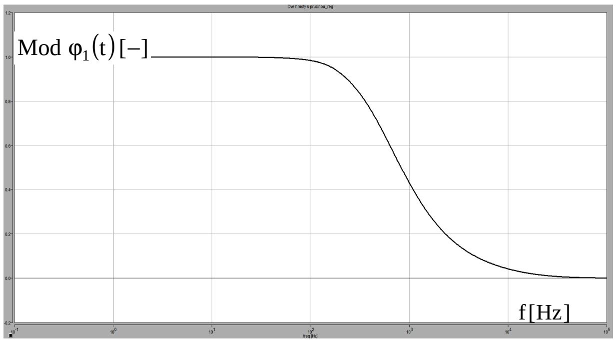

Fig. 4 $\varphi_1(s)$ frequency characteristic

On Fig. 5 the transient characteristic of $\varphi_2(s)$ is seen and on Fig.6 is seen its frequency characteristic.

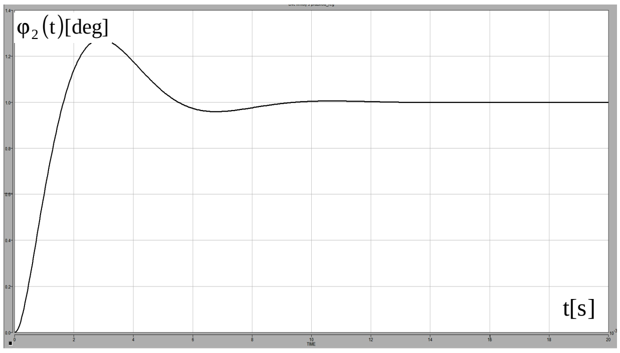

Fig. 5 $\varphi_2(s)$ transient characteristic

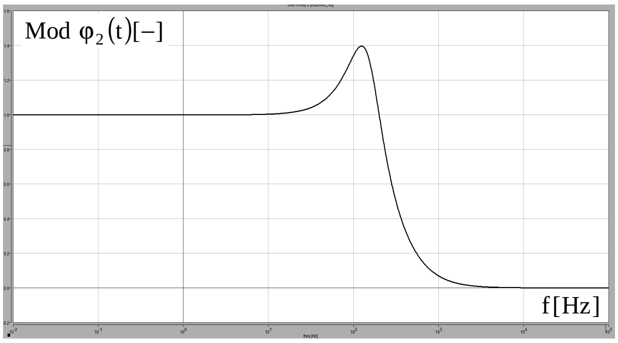

Fig. 6 $\varphi_2(s)$ frequency characteristic

While angle $\varphi_1(s)$ is quickly achieving its reference value without oscillations, angle $\varphi_2(s)$ oscillates through spring and mass $\mathbf{J_2}$ influence.

3 Problem Power Interactions physical Model

Further we present simulation of rotational masses speed control problem globally, whereas at using of problem physical model we'll study nonlinear spring whose dependence **Q=f(Δφ)** is on Fig.7.

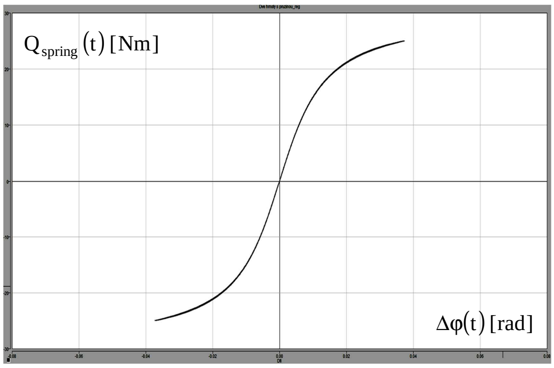

Fig. 7 Nonlinear spring characteristics

Speed controller we think with saturation, so with limitation to range ± 10.

On Fig.8 the power interactions model of analysed system in system Dynast for mechatronics systems simulation and analysis is presented, including nonlinear spring and controlled by P-I regulator with limitation (antiwind-up).

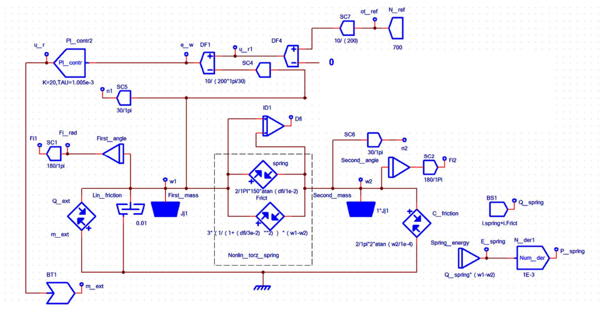

Fig.8 Analysed system power interactions physical model.

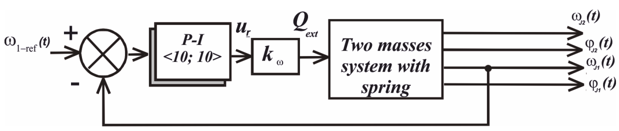

Fig.9 Block diagram of non-damped speed control system

On Fig.9 is block-diagram this non-damped speed control system.

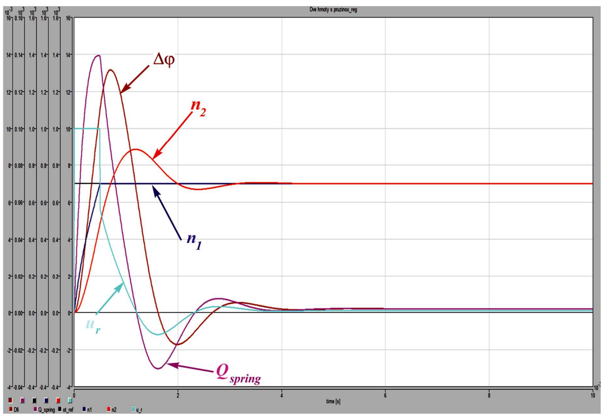

Fig.10 Response on first mass speed requested value jump $n_{1\_ref} = 700 \text{tours/min}$

Complete system simulation acknowledges problem of linear variant analysis. Because information about motion (angular velocity) is measured on the first mass, second mass, connected over spring, oscillates with damping.

3.1. Simple active Damping Principle

Try to solve introduced problem of mass $\mathbf{J_2}$ oscillation connected over spring on the basis of empiric procedure:

First of all pose question, what's mentioned behaviour reason! Is evident, that mass $\mathbf{J_2}$ oscillations and by interaction over spring also mass $\mathbf{J_1}$ oscillations causes just mass $\mathbf{J_2}$, which is in torque interaction with the rest of system over torsional spring.

Are we able to reconstruct somehow these effects, that we could damp them subsequently? Is evident, that on mass $\mathbf{J_1}$ functions partly outer torque $\mathbf{Q_{ext}}$ (reduced by friction in first masses bearings) and this torque we have under control. We are able to determine it from regulator output $\mathbf{u_r}$.

And further functions on this mass the $\mathbf{J_2}$ mass over spring. So total dynamic acting force on mass $\mathbf{J_1}$ is

$$Q_{ext} - b \cdot \omega_1 - Q_{react\_J_2}$$

But this torque has to be in every instant in balance with mass $\mathbf{J_1}$ inertial torque. So:

$$J_1 \cdot \frac{d\omega_1}{dt} = Q_{ext} - \hat{b} \cdot \omega_1 - Q_{react\_J_2}$$

Assume, that we know how estimate moment of inertia $\mathbf{J_1}$ size.

Designate this estimation (measurement, catalogue specification) $\hat{J}_1$. Never mind further reads that we know how „to determine" rotational acceleration $\frac{d\omega_1}{dt}$. Designate it $\hat{\varepsilon}$. Then in every instant reads:

$$Q_{react\_J_2} \approx Q_{ext} - \hat{b} \cdot \omega_1 - \hat{J}_1 \cdot \hat{\varepsilon} \tag{3}$$

By introduction of correction proportional to this reaction to the speed reference value, the active oscillations damping of second mass can be acquire. Then both masses will behave approximately in the same way.

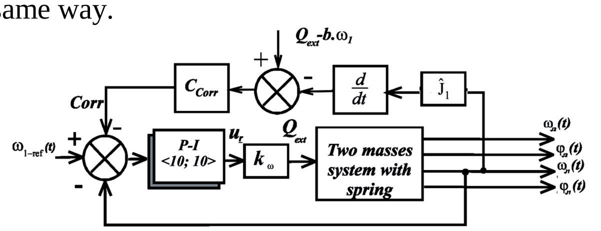

Fig.11 Theoretical block diagram of damped speed control system

One question remains. How perform $\hat{\varepsilon}$ reconstruction in real industrial environment?

3.2. Acceleration Reconstruction in industrial Conditions

Because in real conditions is direct derivation signal generation problematic, perform its following reconstruction:

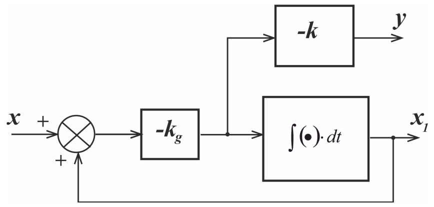

Fig.12 Derivation reconstruction

From Fig.12 follows

$$X_1(s)=\frac{1}{s}\cdot\{-k_g\cdot[X(s)+X_1(s)]\}\Rightarrow$$

$$\Rightarrow Y(s)=-k\cdot\{-k_g\cdot[X(s)+X_1(s)]\}=k\cdot\frac{s}{1+\frac{1}{k_g}s}\cdot X(s)$$

So, for this arrangement reads

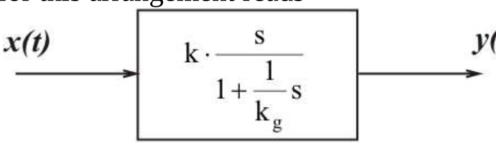

Fig.13 Block of derivation reconstruction

and we obtain derivations with 1^{th} order filter.

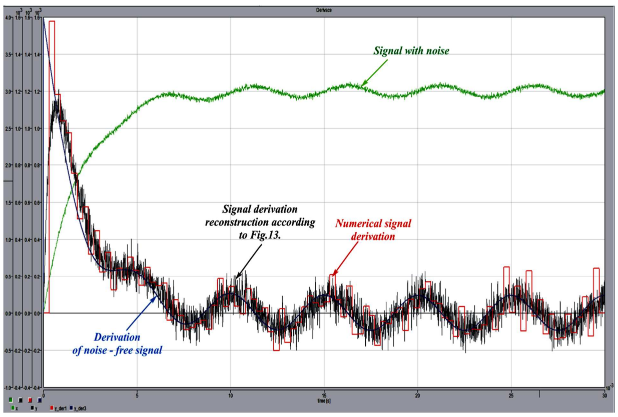

Fig.14 Result of signal with noise derivation

On Fig.14 is see that signal with random signal noise is not simple to differentiate! And how signal with noise mathematical derivation would look?

3.3. Application of simple active Damping

On Fig.15 the power interactions multiport model of analysed system is presented, where the active damping is introduced by means of mass J_1 acceleration reconstruction.

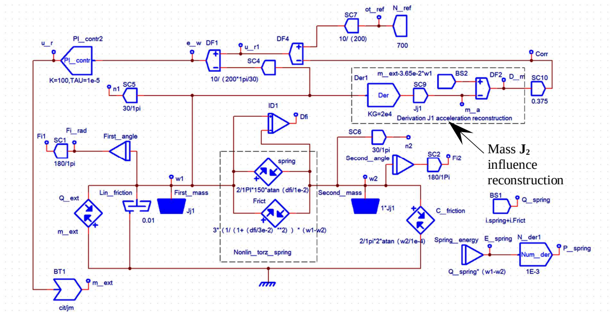

Fig.15. Analysed system power interactions with acceleration reconstruction and active damping

On Fig.16 is response of system with active damping to identical mass $\mathbf{J_1}$ requested speed jump like in Fig. 10.

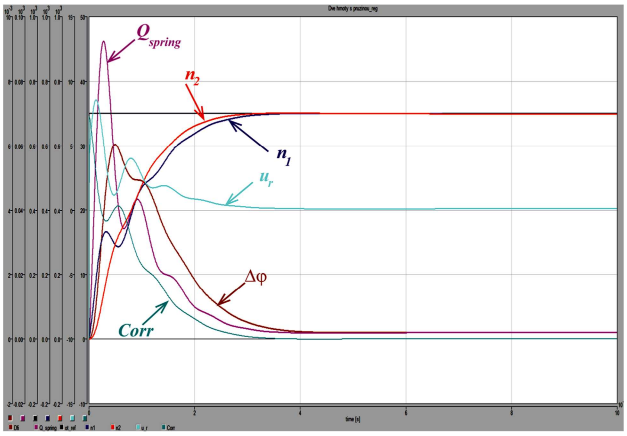

Fig.16. Response on first mass position requested value jump $n_{1\_ref}=700\,\text{tours/min}$ with active damping

Is evident, that whereas in the event of undamped motion, the mass $\mathbf{J_1}$ speed required (and sensing on it) evokes mass $\mathbf{J_2}$ oscillations, in second case mass $\mathbf{J_1}$ „will wait" on mass $\mathbf{J_2}$ and so it is possible to adjust it also at scanning on actuator (engine) mass.

The first part paper shows and gives reasons for one from simple motion control problem solution with scanning on actuator and forms that way starting point to second part of those motion control way of solution.

4 Active Damping with State Regulator

4.1. Description of system with one directional motion and its active damping principle

For linear system from Fig.17, where force **f(t)** is created by actuator according to Fig.18 reads:

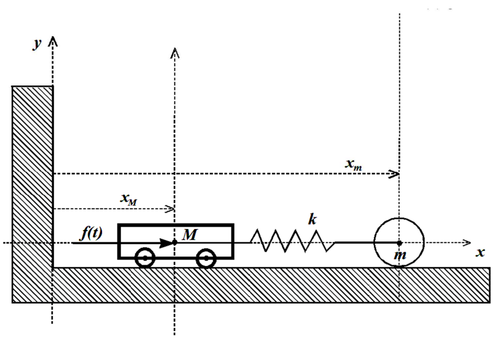

Fig. 17 Principle of system with motion in one direction.

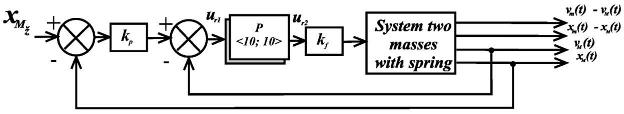

Fig. 18 Masse **M** cascade position control.

$$f(t) = k_f \cdot \left\{ k_{pv} \cdot \left[\overbrace{k_p \cdot (x_{Mž} - x_M)}^{u_{r1}} - 1/200 \cdot v_M \right] \right\} =$$

$$= k_f \cdot k_{pv} \cdot k_p \cdot x_{Mž} - k_f \cdot k_{pv} \cdot k_p \cdot x_M - \frac{k_f \cdot k_{pv}}{200} \cdot v_M$$

and for state model reads:

$$\frac{d}{dt}\begin{bmatrix} v_m - v_M \\ x_m - x_M \\ v_M \\ x_M \end{bmatrix} = \begin{bmatrix} 0 & -\frac{k(M+m)}{M \cdot m} & +\frac{1}{200M} \cdot k_f \cdot k_{pv} & \frac{1}{M} \cdot k_f \cdot k_{pv} \cdot k_p \\ 1 & 0 & 0 & 0 \\ 0 & \frac{k}{M} & -\frac{1}{200M} \cdot k_f \cdot k_{pv} & -\frac{1}{M} \cdot k_f \cdot k_{pv} \cdot k_p \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_m - v_M \\ x_m - x_M \\ v_M \\ x_M \end{bmatrix} + \quad (4)$$

$$+ \begin{bmatrix} -\frac{1}{M} \cdot k_f \cdot k_{pv} \cdot k_p \\ 0 \\ \frac{1}{M} \cdot k_f \cdot k_{pv} \cdot k_p \\ 0 \end{bmatrix} \cdot x_{Mž}$$

$$c^T = [0 \quad 0 \quad 0 \quad 1]$$

If mass **m** is „extended" to **1m** in the positive direction and reference value $x_{M_ž} = 0$, then after mass **m** releasing, the regulators in cascade will ensure almost mass **M** perfect still stand. Mass **m** after releasing practically oscillates without damping. Mentioned on Fig.19 is seeing.

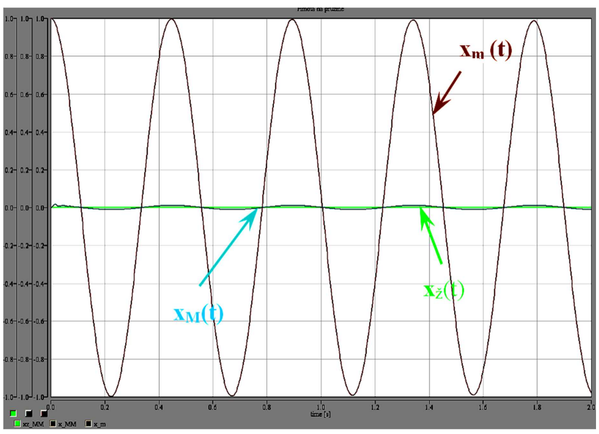

Fig. 19 The mass **m** undamped oscillation and mass **M** stabilization after releasing of extended mass **m**

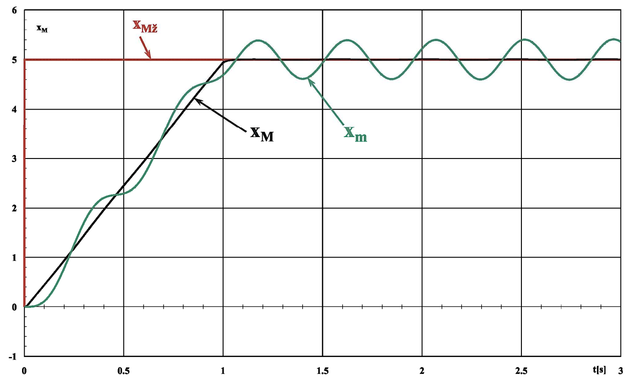

Fig. 20 System (4) response on mass **M** required position jump $\mathbf{x_{Mž} = 5\ m}$

On Fig. 20 is response of both mass seen at required mass **M** position jump **5m**. Again is seen the perfect masses **M** behaviour and masses **m** undamped oscillations.

For $M = 400$ kg ; $m = 1000$ kg ; $k = 2.10^5$ N/m; and $k_f = 2.10^6$ N/1V; $k_p = 3$; $k_{pv} = 3$ we obtain the system poles

$$s_1 = -37.498 + i \cdot 209.99$$
$$s_2 = -37.498 - i \cdot 209.99$$
$$s_3 = -0.00183 + i \cdot 14.064$$
$$s_4 = -0.00183 - i \cdot 14.064$$

If we'll require to obtain the new required poles by help of designed complete state regulator

$$s_1 = -50 ; s_2 = -50$$
$$s_3 = -15 + i \cdot 15$$
$$s_4 = -15 - i \cdot 15$$

then is possible to create this complete state regulator as

$$\underline{r}^T = [0.010444 \quad 0.008333 \quad 0.011667 \quad -0.875] \quad (5)$$

(see Fig.20).

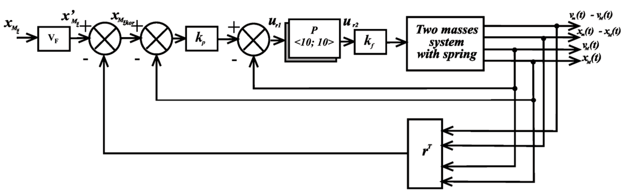

Fig. 21 Complete state regulator

Control structure from Fig.21 will ensure the system behaviour for the same mass **m** „extending„ to **1m** according to Fig.22.

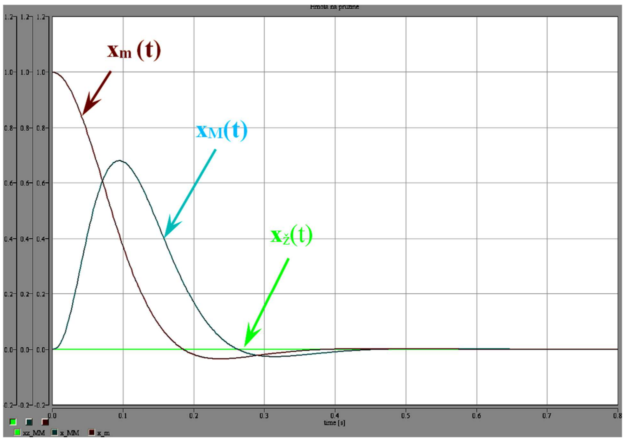

Fig. 22 Complete state regulator damping influence.

It is evident that mass **M** controlled then way suppresses mass **m** oscillations now.

Fig.23 shows such system response on mass **M** desired position jump $\mathbf{x_{M\check{z}} = 5\ m}$.

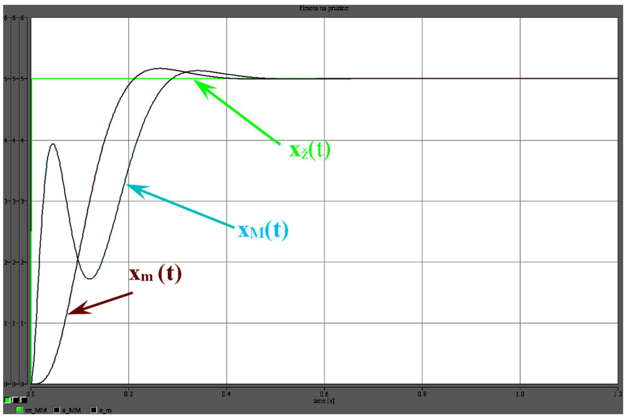

Fig. 23 Response on required **M** position jump with **m** and **M** masses oscillations' active damping

4.2. Linear observer utilisation for active damping of one dimensional system

Measurement of mass **m** position and speed (eventually) presents indeed problem in general.

Because for selected parameters is

$$\mathbf{Q}_p = \begin{bmatrix} \underline{c}^T \\ \underline{c}^T \cdot \mathbf{A} \\ \vdots \\ \underline{c}^T \cdot \mathbf{A}^{n-1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 500 & -50000.0 & -6250.0 \\ 500 & -2.5 \cdot 10^7 & 2.49999375 \cdot 10^9 & 3.125 \cdot 10^8 \end{bmatrix}$$

then

$$\det \mathbf{Q}_p = \det \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 500 & -50000.0 & -6250.0 \\ 500 & -2.5 \cdot 10^7 & 2.49999375 \cdot 10^9 & 3.125 \cdot 10^8 \end{bmatrix} = 250000 \neq 0$$

and the system (4) is observable.

We can design complete linear observer system with select observer matrix eigenvalues

$$s_1 = -50 + 300 \cdot i; \quad s_2 = -50 - 300 \cdot i$$
$$s_3 = -1 + i \cdot 15; \quad s_4 = -1 - i \cdot 15$$

we can obtain the observer equation

$$\dot{\hat{\underline{x}}}(t) = \mathbf{A} \cdot \hat{\underline{x}}(t) + \underline{b} \cdot u(t) + \underline{h} \cdot [y(t) - \hat{y}(t)]$$

$$\dot{\hat{\underline{x}}}(t) = \begin{bmatrix} 0 & -\frac{k(M+m)}{M \cdot m} & +\frac{1}{200M} \cdot k_f \cdot k_{pv} & \frac{1}{M} \cdot k_f \cdot k_{pv} \cdot k_p \\ 1 & 0 & 0 & 0 \\ 0 & \frac{k}{M} & -\frac{1}{200M} \cdot k_f \cdot k_{pv} & -\frac{1}{M} \cdot k_f \cdot k_{pv} \cdot k_p \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \hat{\underline{x}}(t) + \begin{bmatrix} -\frac{1}{M} \cdot k_f \cdot k_{pv} \cdot k_p \\ 0 \\ \frac{1}{M} \cdot k_f \cdot k_{pv} \cdot k_p \\ 0 \end{bmatrix} \cdot x_{M_{\check{z}}}(t) + \begin{bmatrix} -40282 \\ 347.4 \\ 45201 \\ 27 \end{bmatrix} \cdot [x_M(t) - \hat{y}(t)] \tag{6}$$

and we will use only part of reconstructed state quantities from it, so the structure from Fig. 24.

On Fig 25 is seen that active damping with non-measurable variables reconstruction by linear observer give the same result as state controller with full measurable state variables (compare Fig. 23 and Fig. 25).

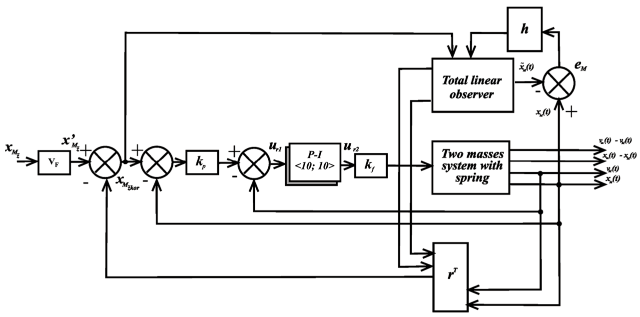

Fig. 24 Active damping with linear observer of **m** and **M** masses

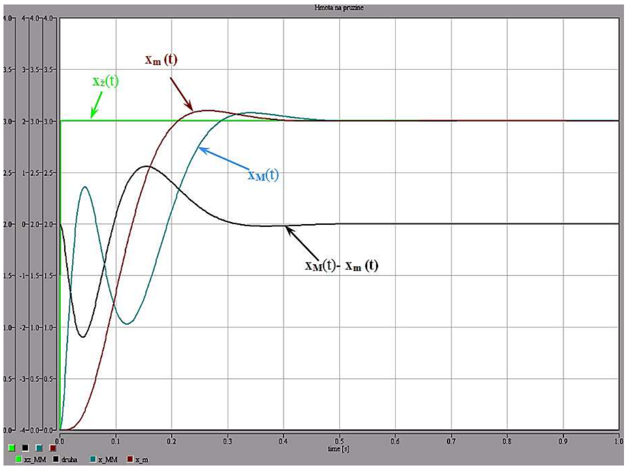

Fig. 25 Response on required **M** position jump with **m** and **M** masses oscillations' active damping with observer utilization

4.3 Active damping of masses motion system in plane

Study ordering from Fig. 26 and Fig 27, so the system with planar motion without friction, normal to gravitation direction, whereas external mass **M** is controlled by e.g. electrohydraulic translational positional servo system producing force **f(t)**. This mass can move only in **x** axis direction.

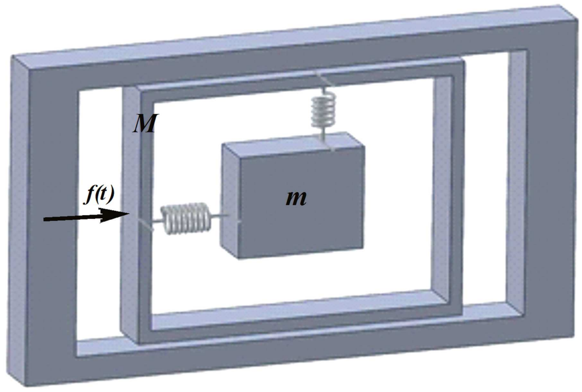

Fig. 26 Ordering of two masses system with four degree of freedom

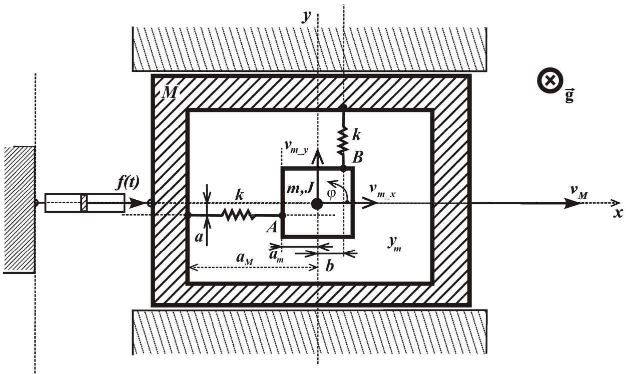

Fig. 27 Two masses system with four degree of freedom scheme

Inside of this material „frame" is mass **m**, „hung" on two linear springs without dissipative damping, whose axes are, in quiescent state (mass **M** and **m** centres of gravity are in identical point, point [0, 0], angle **φ= 0**) displaced in a parallel way from coordinates axes. Mass **m** positive rotation direction is counter-clockwise.

Create physical simulation model of given ordering by means of simulation system for multiport simulation physical models DYNAST [6]:

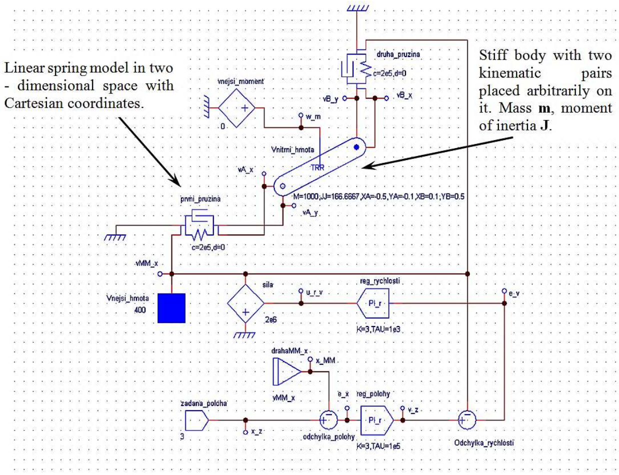

Fig. 28 Multiport physical model of system from Fig.27

„Extend" at first the inner mass **m** to **0.3 m** in positive **x** axis direction, require by frame **M** driving servo to remained this frame quiescent and release mass **m** in time **t = 0**.

On Fig. 29 and Fig. 30 is result of this experiment. Is see, that inner mass **m** oscillates undamped after releasing, whereas the oscillations energy subsequently „overflows" from **x** axis to the **y** axis and inner mass „spins" (angle **φ**).

It's seen that the outer **M** mass fast movement to required position produces not only mass **m** oscillations in **x** axis, but also in **y** axis. In addition, thanks to asymmetric springs bearing, at this mass **m** yawing oscillation happens and successively mechanical energy "flows" between both axes.

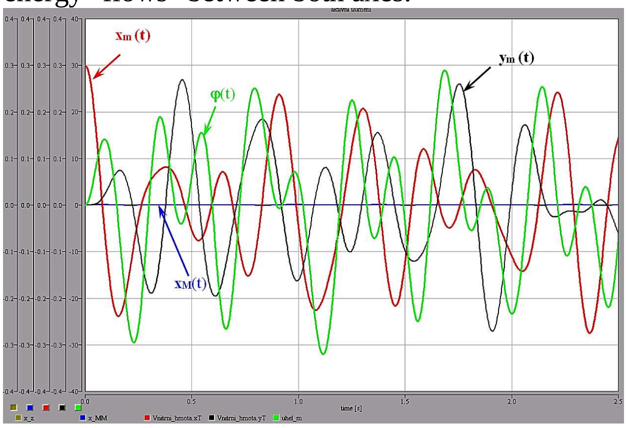

Fig. 29 The mass **m** undamped two dimensional oscillations and mass **M** stabilization after releasing of extended mass **m**

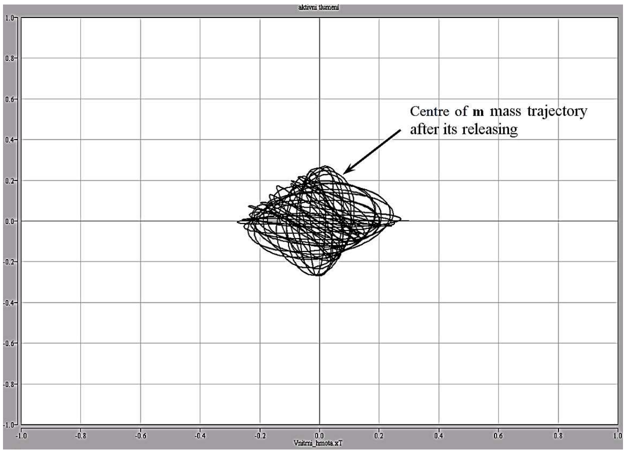

Fig. 30 Centre of mass undamped **m** trajectory in plane for mass **M** stabilization after releasing of extended mass **m**

Fig.31 shows system undamped response on mass **M** required position jump in time **t =0s**; **$x_{Mž}$=3m**.

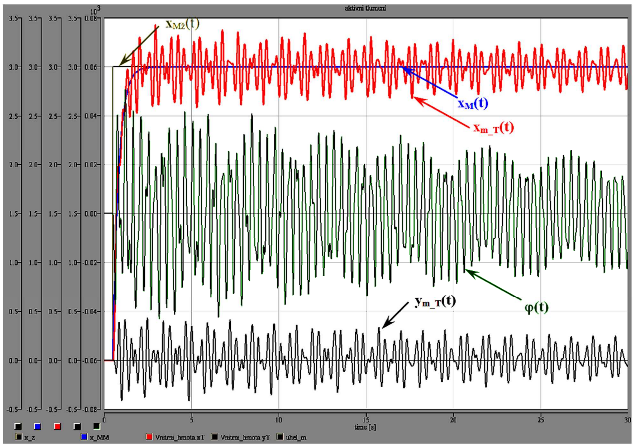

Fig. 31 Undamped system from Fig .26 and Fig.27 behaviour for required **M** mass jump of position

If cuboid side size of mass **m** is equal **2a$_m$**, then motional equations of described system are

$$\begin{aligned}
&M\cdot\ddot{x}_M - k\cdot[(x_T - a_m\cos\varphi + a\sin\varphi - x_M) + (x_T - a_m\sin\varphi + b\cos\varphi - x_M)] = f(t)\\
&m\cdot\ddot{x}_T + k\cdot[(x_T - a_m\cos\varphi + a\sin\varphi - x_M) + (x_T - a_m\sin\varphi + b\cos\varphi - x_M)] = 0\\
&m\cdot\ddot{y}_T + k\cdot[(y_T - a_m\sin\varphi - a\cos\varphi + a) + (y_T + a_m\cos\varphi + b\sin\varphi)] = 0\\
&J\cdot\ddot{\varphi} + k\cdot\begin{bmatrix} a_m\cdot(x_T - x_M)\cdot(\sin\varphi - \cos\varphi) - (x_T - x_M)(b\sin\varphi - a\cos\varphi) + \\ + (y_T + a)(a\sin\varphi - a_m\cos\varphi) - y_T(a_m\sin\varphi - b\cos\varphi)\end{bmatrix} = 0
\end{aligned} \qquad (7)$$

where

x_M is coordinate of **M** mass

x_T, y_T are centre of **m** mass coordinates

φ is angle of **m** mass body with regard to global **x** axis.

Simplify this nonlinear system thinking φ small, so

$$\cos\varphi \approx 1; \sin\varphi \approx \varphi$$

and use cascade position controller with two proportional regulators

$$f(t) = k_f\cdot\left\{k_{pv}\cdot\left[\overbrace{k_p\cdot(x_{Mž} - x_M)}^{v_{Mž}} - 1/200\cdot v_M\right]\right\} =$$

$$= k_f\cdot k_{pv}\cdot k_p\cdot x_{Mž} - k_f\cdot k_{pv}\cdot k_p\cdot x_M - \frac{k_f\cdot k_{pv}}{200}\cdot v_M$$

Then we obtain state system equations of 8th order

$$\frac{d}{dt}\begin{bmatrix} v_{xT}-v_M \\ x_T - x_M \\ v_{yT} \\ y_T \\ \omega \\ \varphi \\ v_M \\ x_M \end{bmatrix} = \begin{bmatrix}
0 & -2\frac{k(M+m)}{M\cdot m} & 0 & 0 & 0 & -\frac{k(M+m)}{M\cdot m}\cdot(a_m - a) & \frac{1}{200M}\cdot k_f\cdot k_{pv} & \frac{1}{M}\cdot k_f\cdot k_{pv}\cdot k_p \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2\frac{k}{m} & 0 & \frac{k}{m}\cdot(a_m - b) & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{k}{m}\cdot(a_m - a) & 0 & \frac{k}{J}\cdot(a_m - b) & 0 & -\frac{k}{J}\cdot a^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 2\frac{k}{M} & 0 & 0 & 0 & \frac{k}{M}\cdot(a_m - a) & -\frac{1}{200M}\cdot k_f\cdot k_{pv} & -\frac{1}{M}\cdot k_f\cdot k_{pv}\cdot k_p \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}\cdot\begin{bmatrix} v_{xT}-v_M \\ x_T - x_M \\ v_{yT} \\ y_T \\ \omega \\ \varphi \\ v_M \\ x_M \end{bmatrix} + \begin{bmatrix} -\frac{1}{M}\cdot k_f\cdot k_{pv}\cdot k_p \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{M}\cdot k_f\cdot k_{pv}\cdot k_p \\ 0 \end{bmatrix}\cdot x_{Mž} + \begin{bmatrix} \frac{k(M+m)}{M\cdot m}\cdot(a_m - b) \\ 0 \\ -\frac{k}{m}\cdot a_m \\ 0 \\ -\frac{k}{J}\cdot a_m\cdot a \\ 0 \\ -\frac{k}{M}\cdot(a_m - b) \\ 0 \end{bmatrix} \qquad (8)$$

With regard of this both material bodies linear description and their possible motion control way with complete linear observer, simplify problem and use only **x** axis for active damping.

It's withal whole rows of real motional systems possibility, when we have available not only measurement at limited points, but also limited operational intervention.

Employ piece of knowledge from previous one-dimensional case. Design complete state regulator for control and active damping in **x** axis and subsequently propose linear observer for $x_{Tm} - x_M$ and $v_{Txm} - v_{xM}$ reconstruction. It means, we suppose that we are able to measure mass **M** position and speed in **x** axis and differences $x_{Tm} - x_M$ and $v_{Txm} - v_{xM}$ we will obtain from observer.

Require the same mass **M** jump as in Fig. 31, but with above mentioned active dumping with state regulator and observer in **x** axis. On Fig. 32 is seen that system mechanical behaviour is damped. Not perfectly, because we dumped only part of energy.

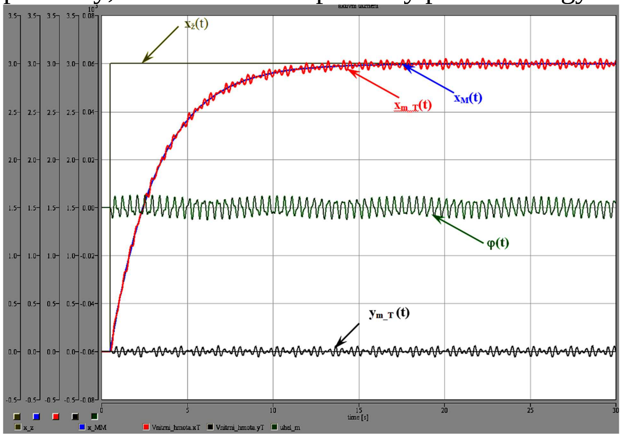

Fig. 32 Behaviour of system from Fig. 26 and Fig. 27 damped in one axis at **M** position desired jump

5 Conclusion

Paper shows controlled mechanical systems' active damping simple possibilities, applicable in case those state variables measurement is made on actuator and the action interventions are available only in one axis. Designed one-dimensional complete state regulator is able- thanks „energy overflow" of unsymmetrical embedded springs- to damp significantly also oscillations of mass, which it is impossible to influence directly by action quantities.

This is frequent problem at mechanical system motion precision with speed or position control by actuator formed by electric or hydraulic engines and it is consequence of fact that controlled value sensing is made commonly on motor, which causes that mechanical part, which is our interest subject, is controlled indirectly.

This problem is presented significantly in moment when between actuator mass (electric machine) and load mass isn't ideally rigid connection. It's in some detail common problem of any transmissivity arrangement containing e.g. the play, but especially significantly such system behavior demonstrates in case of e.g. harmonics gearbox utilization.

This article forms some simplified starting point for next part, which solves mentioned problem by means of full active damping with incomplete observer.

In second part some pieces of knowledge relevant to active damping are introduced. There are the knowledge relevant to active damping of mechanical systems with more degree of freedom with limited action interventions' possibilities and limited or complicated quantities measurement possibility.

This problem often occurs at different mechanical systems motion control types serving as optical (surveillance) or other systems porter, which depend on effector systems positional state accuracy, and when actuators functions in some generalized coordinates only.

As such system example can be cameras porter, laser scanning system porter created from no ideally stiff bodies, weapons porter system with uncontrolled projectiles, but also manipulator with no ideally stiff arms for exact assembly application, not to mention, for invasive medicine application

References:

[1] B. Friedland, *Control system design: An introduction to state-space methods*. *McGraw-Hill*, New York (1986)

[2] L. Zboray, *Vybrané kapitoly z teorie riadenia* Edičné stredisko VŠT v Košiciach (1985) (In Slovak language)

[3] Z. Úředníček, *Robotika*, T. Bata university in Zlin (in Czech language), (2012)

[4] H. A. Darweesh, M. Roushdy, H. M. Ebied, B.M. Elbagoury, Design a cost effective non-holonomic vehicle for autonomous driving applications, *Proceedings of the WSEAS International Conference Mathematical Application, in Science and Mechanics*. (2013)

[5] S. Hubalovsky, P. Kadlec, L. Mitrovic, P. Hanzalova, Computer simulation model of static mechanical properties of real technical device - elevator cab, *Proceedings of the 3rd International Conference on Mathematical Models for Engineering Science (MMES '12)*. Paris, France, (2012)

[6] https://sites.google.com/site/dynasthelp/ (20.7.2017)

[7] H. Mann, Z. Urednicek, A two-level control-design methodology and a software toolset for mechatronics, *IUTAM symposium on interaction between dynamics and control in advanced mechanical systems*, Book Series: Solid Mechanics and its Applications, Volume: 52 Pages: 223-230, Eindhoven, Netherlands, 1997

[8] Z. Úředníček, M. Opluštil, Equations of Motion and Physical Model of Quadcopter in Plain, *Proceedings of the WSEAS International Conference*, CSCC 2014, Volume I, pp. 66-70, ISBN 978-1-61804-234-9, Santorini, Greece

[9] Z. Úředníček, Stabilization of telescopic inverse pendulum verification by physical models, *International journal of mechanics,* **10**, (2016)

[10] Z. Úředníček, R. Drga, Measuring robot kinematics description and its workspace, *MATEC Web of Conferences 76, 02027* (2016)

[11] Jian Mao, Huawen Zheng, Yuanxin Luo, A new Velocity Estimator for Motion Control Systems, *Proceedings of WSEAS Transactions on Systems and Control*, ISSN E-ISSN: 1991-8763/2224-2856, Volume 9, 2014, pp. 209-214.

[12] I. Astrov, M. Pikkov, R. Paluoja., Motion Control of Vectored Thrust Aerial Vehicle for Enhanced Situational Awareness. *Proceedings of the WSEAS International Conference on Mathematical Applications in Science and Mechanics*. Dubrovnik, Croatia, June 25-27, 2013