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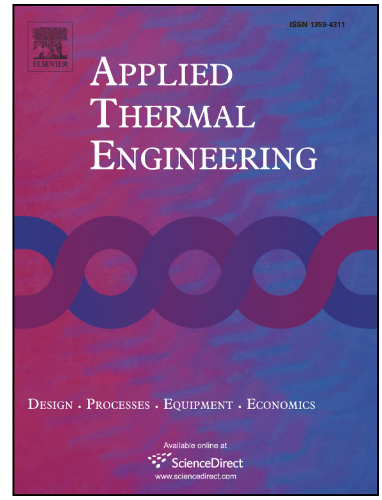

Research Paper

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Robust Stability of Thermal Control Systems with Uncertain Parameters: The Graphical Analysis Examples

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Abstract: This paper is intended to present the investigation of robust stability for integer order or fractional order feedback control loops affected by parametric uncertainty and time-delay(s) with special emphasis on the thermal control systems. The applied graphical method is based on the numerical calculations of the value sets and the zero exclusion condition. Three robust stability examples inspired by control of the real-world thermal processes are used for demonstration of the technique applicability. Namely, the work deals with the analysis of a shell-and-tube heat exchanger which was identified as the (integer order) time-delay model with parametric uncertainty, a heat transfer process modeled as the fractional order time-delay plant with parametric uncertainty, and a heating-cooling system with a heat exchanger described by the anisochronic model with internal delays and parametric uncertainty.

Keywords: Robust Stability Analysis, Thermal Systems, Parametric Uncertainty, Time Delay, Fractional Order Systems.

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1. Introduction

Thermal systems arise in many engineering fields such as manufacturing, power generation, automotive and aerospace engineering, air conditioning, etc. [1] and so their efficient and low-cost control represent an important task. Beyond a doubt, stability is the most critical property of (not only thermal) control systems. Thus, the investigation of the stability is widely studied discipline, which is going to be nontrivial especially if the control systems are affected by phenomena such as nonlinear behavior [2], [3], time delays [4] – [7] or if they are described by means of fractional order derivatives and integrals [8] – [12]. Naturally, these factors can be often mixed and interconnected [13] – [15]. Moreover, the mathematical models of the controlled systems almost never exactly match their real-world behavior due to many reasons, e.g. linearization in an operating point, imperfections in modeling (neglect of nonlinearities, "slow" time-variations, fast dynamic effects, etc.), changes in physical parameters or other perturbations. Respecting these influences in the mathematical description of the systems can lead to the use of the uncertain models and application of robust control [16] – [18]. Such uncertain model can be defined in more ways of which the parametric [19] – [22] and unstructured [23] – [26] approaches belong among the most commonly used ones. Anyway, if the control system is requested to remain stable for all possible members of the family defined via the uncertain model, one speaks about the robust stability.

The great interest of many researchers in robust stability analysis for systems with parametric uncertainty was boosted especially by the famous Kharitonov Theorem [27], or actually after its "rediscovery" for a wider automatic control community e.g. in [28]. In spite of the fact that Kharitonov Theorem represents the real milestone in the parametric robust control, its application is limited for the interval polynomials with mutually independent uncertain coefficients. However, the coefficients are often mutually dependent in practical systems and thus the tools for robust stability analysis of such systems were developed. The Edge Theorem, the 32 Edge Theorem (or similar Generalized Kharitonov Theorem), 16 Plant Theorem or Mapping Theorem belong among the best-known ones [16], [17], [21]. Their selection depends mainly on the level of dependency among the coefficients or other specific conditions. The research attention was paid also to the robust stability of time-delay systems [29] – [32] which can lead to the analysis of families of quasi-polynomials. Furthermore, the fractional order systems with parametric uncertainty have come under the spotlight recently [33] – [48]. Compared with the other methods, the graphical approach utilized in this paper, which is based on the value set concept and the zero exclusion condition [16], [19], is relatively easy-to-use and it is particularly very universal, i.e. it can be applied from the simplest up to the very complicated uncertainty structures, including the quasi-polynomial families and fractional order cases. Moreover, the robust stability results are obtained without any conservatism (with the necessary and sufficient condition). On the other hand, a long computational time for a high number of uncertain parameters can be considered as the weakness of the technique.

As it has been already adumbrated, the thermal systems, as a complex class of plants and processes, are generally burden with all the mentioned negative effects, which makes their control complicated. There are various techniques for overcoming these obstacles in thermal systems control available. They include e.g. robust control [49], [50], (robust) model predictive control [51], adaptive control [52], artificial neural networks, fuzzy control [53] and many others. Obviously, these methods are often interconnected as well. As the application of fractional calculus has become extremely popular in various engineering areas [11] recently, the thermal systems are no exception [54] – [57].

This paper presents the robust stability analysis for closed control loops with a fixed integer or fractional order controller (or two feedback controllers) and integer or fractional order controlled plant with parametric uncertainty and time-delay(s) with the special accent on the thermal control systems. The utilized graphical technique is based on the numerical calculations of the value sets and the application of the zero exclusion condition [16], [19]. In order to show the usability of the method, three robust stability examples inspired by control of the real-world thermal processes are elaborated. Specifically, the paper contains the robust stability analysis of the feedback control loops with:

- A shell-and-tube heat exchanger which was identified as the (integer order) time-delay model with parametric uncertainty [50],
- A heat transfer process modeled as the fractional order time-delay plant with parametric uncertainty [56], and
- A heating-cooling system with a heat exchanger described by the anisochronic model with internal delays and parametric uncertainty [58].

To the best of authors' knowledge, the graphical analysis of robust stability for a loop with an anisochronic model with internal delays and uncertain parameters, presented in the last example, was not previously published to date.

This brief paper is organized as follows. In Section 2, the theoretical foundations of a graphical approach to robust stability analysis for integer order or fractional order systems with parametric uncertainty are presented. The key Section 3 then shows three graphical analysis examples for the thermal control systems with, successively, a shell-and-tube heat exchanger [50], a heat transfer process [56], and a heating-cooling system with a heat exchanger [58]. Finally, Section 4 offers some conclusion remarks.

2. A Graphical Approach to Robust Stability Analysis under Parametric Uncertainty

The robust stability of the family of closed-loop thermal systems will be investigated through the robust stability of the family of its closed-loop characteristic polynomials (or to be more precise, retarded quasi-polynomials within this paper).

The family of continuous-time (fractional order or integer order) (quasi-)polynomials is [16]:

$$P = \{ p(s,q) : q \in Q \} \tag{1}$$

where q is the vector of uncertainty, $p(s,q)$ is a (quasi-)polynomial with parametric uncertainty, and Q is the uncertainty bounding set. Most commonly, Q is assumed as a multidimensional box, which means that individual components of vector q are bounded by intervals. The family of (quasi-)polynomials (1) is robustly stable if and only if $p(s,q)$ is stable for all $q \in Q$.

Even for the "ordinary" integer order families of polynomials, the robust stability analysis can represent a nontrivial task. The critical factor for the decision on a convenient tool for investigation can be seen in the complexity of the coefficient functions in the polynomial $p(s,q)$. Generally, the more complicated relations among coefficients of the polynomial means the more complicated robust stability analysis. A classification of the uncertainty structures with growing generality (and complexity) and some typical tools for their robust stability testing can be found e.g. in [16], [19], [20], [59].

The situation is even more complicated for the fractional order systems. However, several approaches have been already developed and the number is still growing [33] – [48].

The examples of thermal systems in this paper lead to various types of (integer order or fractional order) families of retarded quasi-polynomials. Some relevant methods are also available in the literature – e.g. [29] – [32].

Nonetheless, one graphical method seems to be unique from the viewpoint of its universality. It is based on the combination of the value set concept and the zero exclusion condition [16]. It can be applied to a wide range of uncertainty structures, from the simplest to very complicated ones, which suffer from the lack of suitable techniques. Furthermore, it is applicable also for various regions of stability (so-called robust *D*-stability). More details on parametric uncertainty and robust stability analysis and also examples of the typical value sets for the integer order systems can be found in [16] and subsequently e.g. in [19], [20]. The works [33], [34], [38], [39], [43], [44] extended the idea of the value set concept also to fractional order uncertain polynomials and the papers [59], [48] showed their use for the specific class of families of fractional order uncertain quasi-polynomials.

According to the definition, the value set for the family of (quasi-)polynomials (1) at the frequency $\omega \in \mathbb{R}$ is [16]:

$$p(j\omega, Q) = \left\{ p(j\omega, q) : q \in Q \right\} \tag{2}$$

which means that $p(j\omega, Q)$ is the image of *Q* under $p(j\omega, \cdot)$. In practice, the value sets can be constructed by substituting *s* for $j\omega$, fixing ω and letting the vector of uncertain parameters *q* range over the set *Q*.

The zero exclusion condition for the (Hurwitz) stability of the family of continuous-time (quasi-)polynomials (1) can be formulated [16]: Assume invariant degree of (quasi-)polynomials in the family, pathwise connected uncertainty bounding set *Q*, continuous coefficient functions, and at least one stable member $p(s, q^0)$. Then the family *P* is robustly stable if and only if the origin of the complex plane (zero point) is excluded from the value set $p(j\omega, Q)$ at all frequencies $\omega \geq 0$, i.e. P is robustly stable if and only if:

$$0 \notin p(j\omega, Q) \quad \forall \omega \geq 0 \tag{3}$$

In some specific cases (both for the integer and fractional order families), the value sets can be constructed on the basis of analysis of vertices and exposed edges [34], [38], [39], [43], [44].

In this paper, the value sets for the families with relatively complicated uncertainty structures are plotted by using sampling (gridding) the uncertain parameters and subsequent direct calculation of related partial points of the value sets for a supposed frequency range. In other words, a suitable set of non-negative frequencies is pre-selected, then the value set for each frequency is plotted and all those individual value sets are practically composed of the points corresponding to the images of all variations of the appropriately sampled uncertain parameters. Once the applicable value sets are obtained, their position in relation to the complex plane origin has to be checked. As mentioned above, the family is robustly stable if and only if the zero point is excluded from the value sets and all required preconditions are fulfilled, especially the existence of at least one stable member of the family. Actually, the existence of a stable member could be verified before the analysis itself and if the chosen member is found unstable, the graphical test can be skipped because the whole family is robustly unstable. The relative simplicity and applicability also for the complex uncertainty structures represent the main advantage of this approach. Moreover, the robust stability is always tested with the necessary and sufficient condition, even for these complicated uncertainty structures. On the other hand, a long computational time for a high number of uncertain parameters is the weakness. Naturally, the potential complications can be caused by wrong identification and consequently by the incorrect construction of the model with parametric uncertainty which then inadequately describes the real system. However, it is a general problem and a deeper analysis of the identification issues is beyond the terms of this manuscript.

3. Illustrative Examples

This Section is devoted to the specific examples of robust stability investigation for the real thermal systems adopted from the literature. Remind that, despite the different types of the systems in the individual examples, all three examples use the same graphical approach described in Section 2, which is enabled by the above-mentioned universality of the technique. The three Subsections are focused on the analysis of the feedback control circuits with, successively:

- A shell-and-tube heat exchanger [50],
- A heat transfer process [56], and
- A heating-cooling system with a heat exchanger [58].

From the control systems theory viewpoint, the analyzed controlled processes are described by, sequentially:

- The (integer order) time-delay model with parametric uncertainty,
- The fractional order time-delay model with parametric uncertainty, and
- The anisochronic model with internal delays and parametric uncertainty.

3.1 Example 1 – A Shell-and-Tube Heat Exchanger (Integer Order Time-Delay Model with Parametric Uncertainty)

In the first example, consider a counter-current shell-and-tube heat exchanger where petroleum is heated by hot water passing through a copper tube. More details on this heat exchanger, its mathematical description, and various approaches to its control can be found in

[49], [50], [53]. The hot water flow rate and the outlet petroleum temperature were selected as the control input and controlled output, respectively. Originally nonlinear and complex dynamics of the heat exchanger can be incorporated into a simplified model with parametric uncertainty. In [50], the process was identified and described by the second order time-delay plant with the uncertain parameters that can vary within supposed intervals:

$$G(s,K,T,\Theta)=\frac{K}{(Ts+1)^2}e^{-\Theta s}=\frac{[37000,\,70000]}{([20,\,30]s+1)^2}e^{-[0.2,\,2.8]s} \tag{4}$$

This linear model with parametric uncertainty is supposed to cover the true nonlinear process with asymmetric dynamics including potential other perturbations, parameter variations, measurement noise, etc.

More controllers were designed for a fixed nominal system $G(s)=53500e^{-1.5s}\big/(25s+1)^2$ in [50]. The H_2/H_∞ controller, which was reported as a reasonable compromise among controller design, structure and performance, is adopted for the tests in this paper. Such controller was designed in [50] as:

$$C(s)=\frac{0.5s^2+0.05s+0.013}{s^3+7.2s^2+25s+0.32} \tag{5}$$

The corresponding family of closed-loop characteristic quasi-polynomials is:

$$\begin{aligned}&p_{CL}(s,K,T,\Theta)=T^2s^5+\left(7.2T^2+2T\right)s^4+\left(25T^2+14.4T+1\right)s^3+\cdots\\&+\left(0.32T^2+50T+7.2\right)s^2+\left(0.64T+0.25\right)s+0.32+Ke^{-\Theta s}\left(0.5s^2+0.05s+0.013\right);\\&K\in[37000,\,70000],\,T\in[20,\,30],\,\Theta\in[0.2,\,2.8]\end{aligned} \tag{6}$$

The value sets of the family (6) for frequencies from 0 to 3 with the step 0.005 are plotted in Fig. 1. At each frequency, all 3 uncertain parameters are sampled by 30 equidistant values, i.e. each value set consist of $30^3=27000$ points. The full view from Fig. 1 indicates the inclusion of the complex plane origin in the value sets. A zoomed version of the same plot in

Fig. 2 just confirms this fact. It means that the family (6), and thus also the feedback control loop with the plant (4) and controller (5), is robustly unstable, i.e. it becomes unstable for some variations of the possible uncertain parameters from (4).

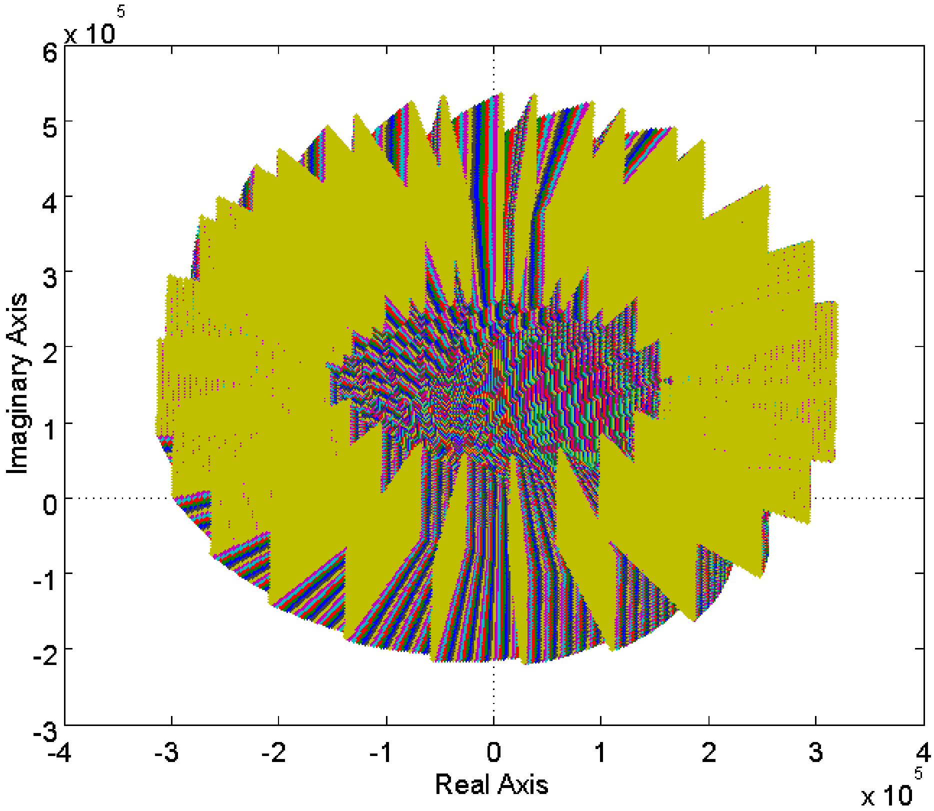

Fig. 1 – The value sets of the family (6) – the full view

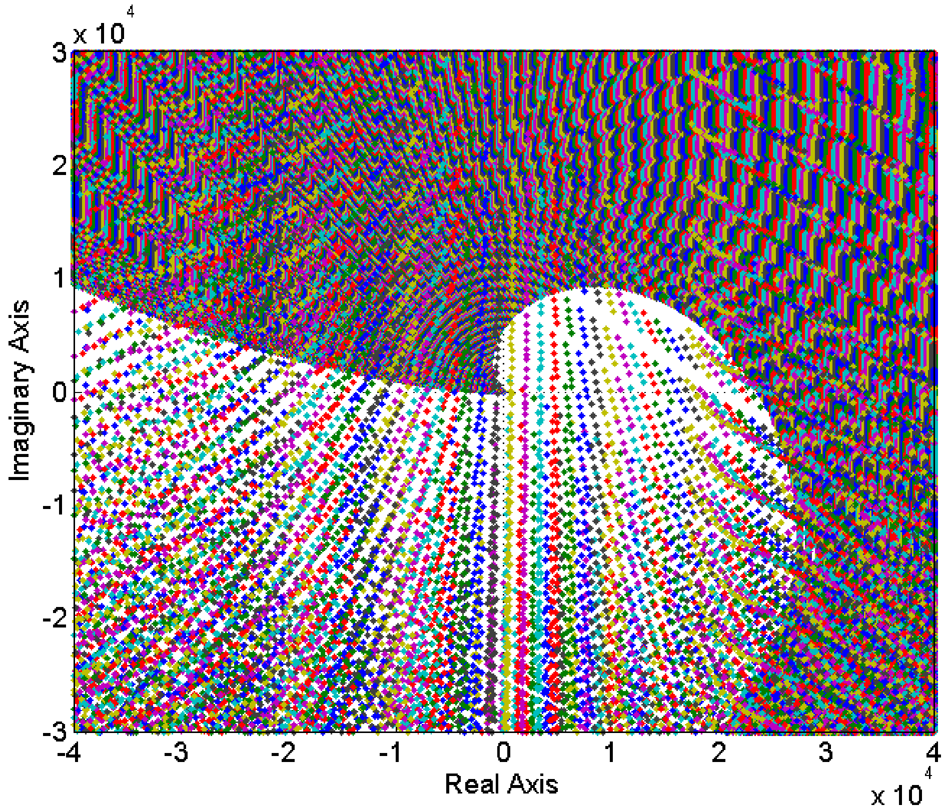

Fig. 2 – The value sets of the family (6) – a zoomed view

Since the adopted controller (5) does not ensure robust stabilization of the system (4), the test is repeated for the narrowed boundaries of the uncertain parameters:

$$K \in [53000, 54000], T \in [24, 26], \Theta \in [1.45, 1.55] \tag{7}$$

For this case, the value sets are plotted for the frequencies from 0 to 10 with the step 0.02. The full view is shown in Fig. 3 and a closer look near the zero point is depicted in Fig. 4. The family contains a stable member [60] and, as can be seen, the origin of the complex plane is excluded from the value sets. It means the family of closed-loop characteristic quasi-polynomials with the structure from (6) and the new uncertain parameters (7) is robustly stable.

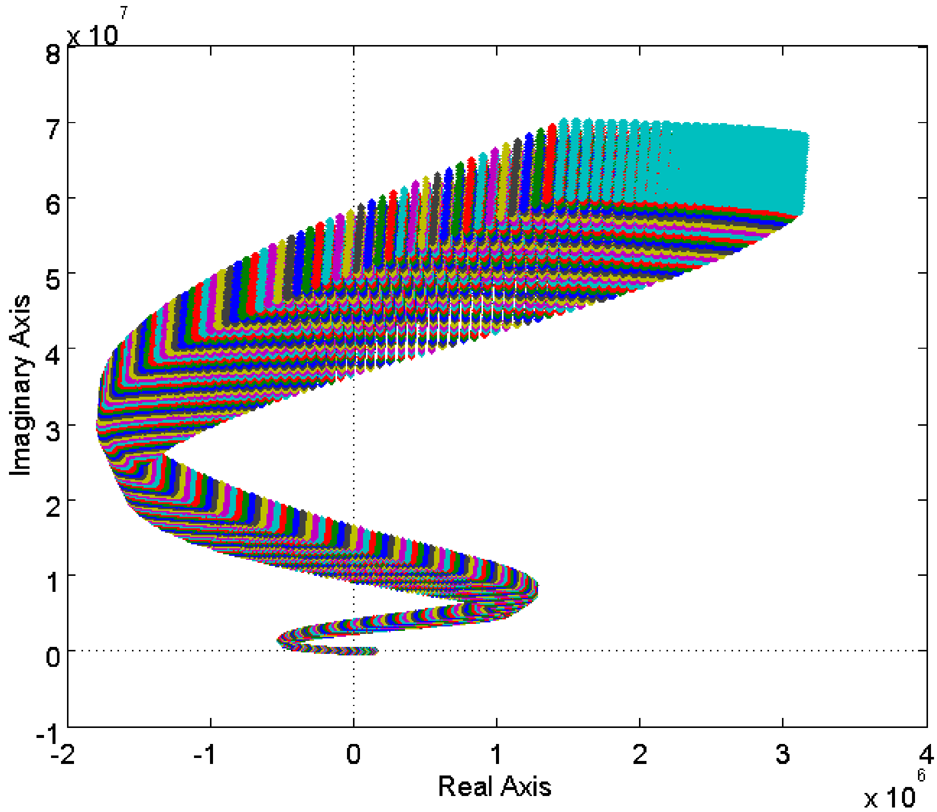

Fig. 3 – The value sets of the family (6) with parameters (7) – the full view

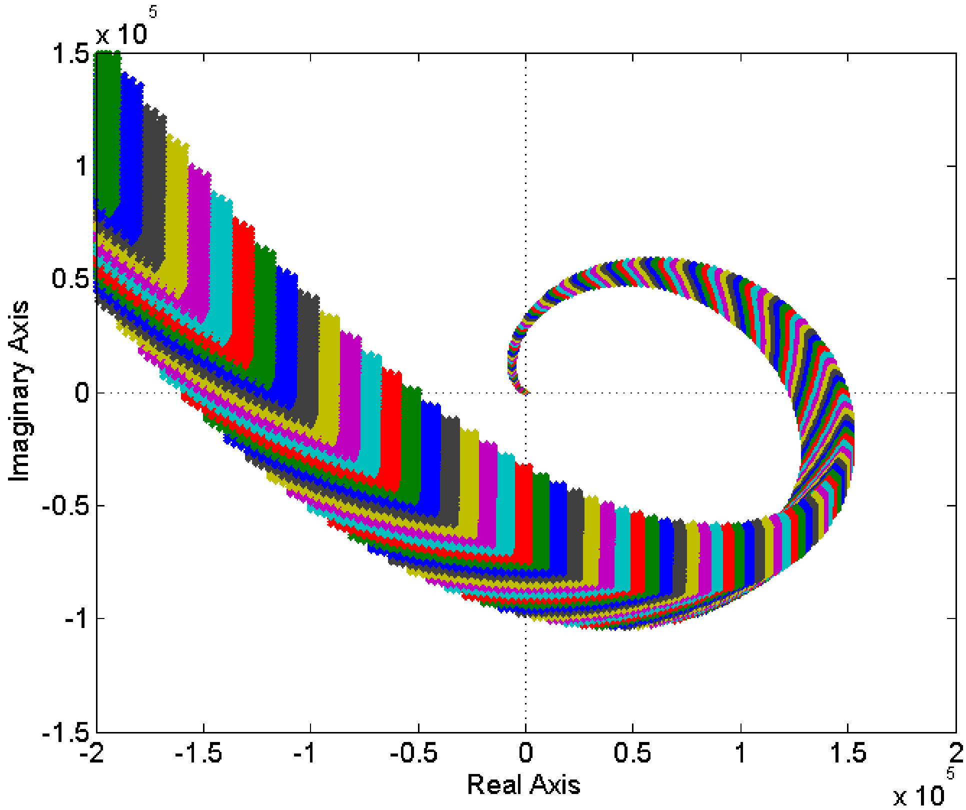

Fig. 4 – The value sets of the family (6) with parameters (7) – a zoomed view

3.2 Example 2 – A Heat Transfer Process (Fractional Order Time-Delay Model with Parametric Uncertainty)

In the second example, heat transfer process for the heat flux loss case, which was mathematically described by means of the fractional order partial differential equation in [56], [57], is supposed. The transfer function describing the temperature at the point λ_1 with respect to the heat flux at the beginning of the beam was derived, identified and verified in [56], [57] as well. This transfer function is:

$$G(\lambda_1, s) = \frac{T(\lambda_1, s)}{H(0,s)} = \frac{K}{T_1 s^{0.5} + 1} e^{-\Theta s^{0.5}} \tag{8}$$

where the experimentally obtained parameters from [56] are $K = 3.2$, $T_1 = 209.27$ and $\Theta = 2.97$.

Assume that the true values of the parameters from (8) are not known exactly but they can vary within the range ±10% of the nominal values, i.e. the heat transfer process is described by the uncertain fractional order time-delay plant:

$$G(\lambda_1, s, K, T_1, \Theta) = \frac{T(\lambda_1, s)}{H(0,s)} = \frac{[2.88,\ 3.52]}{[188.343,\ 230.197]\, s^{0.5} + 1} e^{-[2.673,\ 3.267] s^{0.5}} \tag{9}$$

In [56], several integer and fractional order PID controllers were designed for the nominal system (8) and compared mutually. The fractional order PID controller:

$$C(s) = 259.45\left(1 + \frac{1}{67.74 s^{0.5}} + 0.18 s^{0.5}\right) \tag{10}$$

with fixed integration and differentiation orders 0.5 and with the parameters obtained by minimization of ITAE criterion seems to represent a very good compromise between the performance indicators and minor steady state oscillations [56].

The family of closed-loop characteristic fractional order quasi-polynomials, which corresponds to the feedback control system with the plant (9) and controller (10), is:

$$p_{CL}(s,K,T_1,\Theta) = 67.74T_1 s + 67.74s^{0.5} + Ke^{-\Theta s^{0.5}}\left(3163.52574s + 17575.143s + 259.45\right);$$
$$K \in [2.88, 3.52], T_1 \in [188.343, 230.197], \Theta \in [2.673, 3.267] \tag{11}$$

The Fig. 5 shows the value sets of the family (11) for the frequencies from 0 to 10 with the step 10^{-4}. Each value set consist of $5^3 = 125$ points as all 3 uncertain parameters are sampled by 5 equidistant values. The complex plane origin is excluded from the value sets and, moreover, the family contains a stable member, which can be verified by the generalization of modified Mikhaylov stability criterion from [61]. Thus, the family (11) is robustly stable, i.e. the fractional order PID controller (10) stabilizes the heat transfer process (9) for all possible parameter variations.

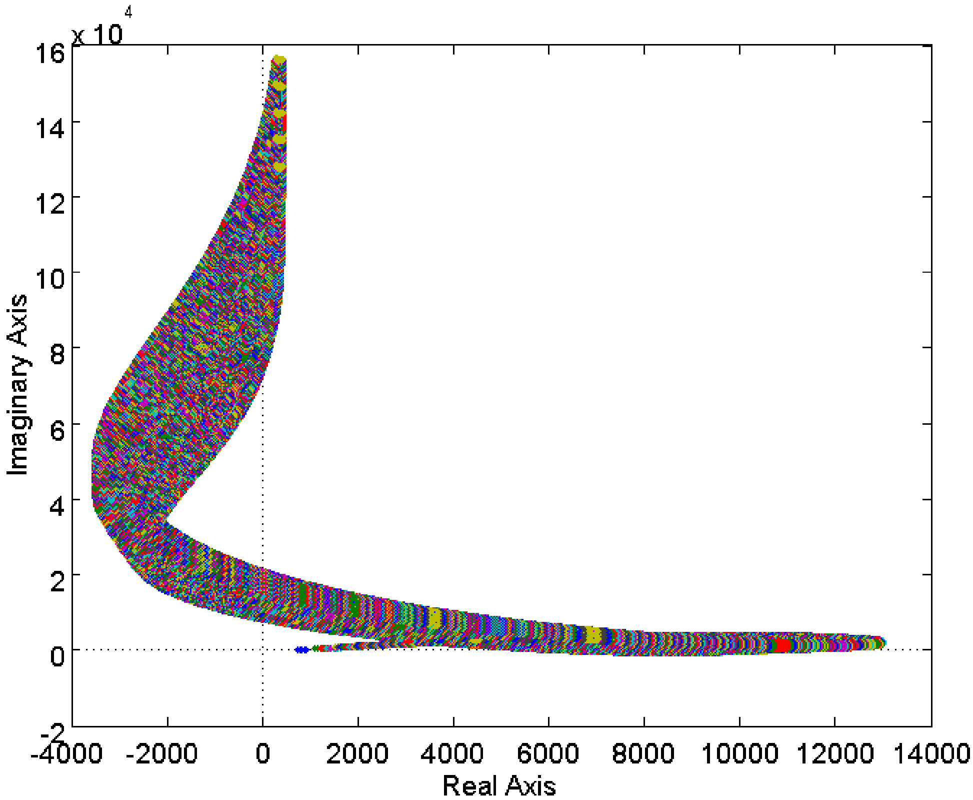

Fig. 5 – The value sets of the family (11)

3.3 Example 3 – A Heating-Cooling System with a Heat Exchanger (Anisochronic Model with Internal Delays and Parametric Uncertainty)

The last example is intended for analyzing a cooling-heating plant with a through-flow air-water heat exchanger that evinces long internal delays. This system was modeled e.g. in [58], [62] on the basis of the anisochronic principle [63] including all the significant delays and latencies in the model caused by the heat transferring from a source through a piping system by using the heat transferring media (water) into a heat-consuming part. In [58], [62], the nonlinear process is described by a set of ordinary differential equations and algebraic equations. The same works derived also a linearized model of the relation between the input power to the heater and the outlet stream temperature from the heat exchanger, which can be formulated by the transfer function:

$$G(s) = \frac{\left(b_0 + b_{0D} e^{-\Theta_0 s}\right) e^{-\Theta_b s}}{s^3 + a_2 s^2 + a_1 s + a_0 + a_{0D} e^{-\Theta_a s}} \tag{12}$$

The identified nominal parameters in the specific operating point are [58]:

$$\begin{aligned} b_0 &= -2.052 \times 10^{-7} \\ b_{0D} &= 2.334 \times 10^{-6} \\ \Theta_0 &= 1.5 \\ \Theta_{0b} &= 141 \\ a_2 &= 0.1767 \\ a_1 &= 0.08989 \\ a_0 &= 1.413 \times 10^{-4} \\ a_{0D} &= -7.625 \times 10^{-5} \\ \Theta_{0a} &= 151 \end{aligned} \tag{13}$$

The measurements in [58] exposed that the process is suitable for robust control since there are perturbations in model parameter values as well as many other non-modeled internal and external influences. For the purpose of analysis in this paper, the possible variations of the size ±10% of all the nominal parameter values are assumed, i.e.:

$$
\begin{aligned}
& b_0 \in \left[-2.2572 \times 10^{-7}, -1.8468 \times 10^{-7} \right] \\
& b_{0D} \in \left[2.1006 \times 10^{-6}, 2.5674 \times 10^{-6} \right] \\
& \Theta_0 \in [1.35, 1.65] \\
& \Theta_{0b} \in [126.9, 155.1] \\
& a_2 \in [0.15903, 0.19437] \\
& a_1 \in [0.080901, 0.098879] \\
& a_0 \in \left[1.2717 \times 10^{-4}, 1.5543 \times 10^{-4} \right] \\
& a_{0D} \in \left[-8.3875 \times 10^{-5}, -6.8625 \times 10^{-5} \right] \\
& \Theta_{0a} \in [135.9, 166.1]
\end{aligned} \tag{14}
$$

The structure with two feedback controllers [64], [65] is used as a suitable control loop for the plant. The diagram of this control configuration is depicted in Fig. 6.

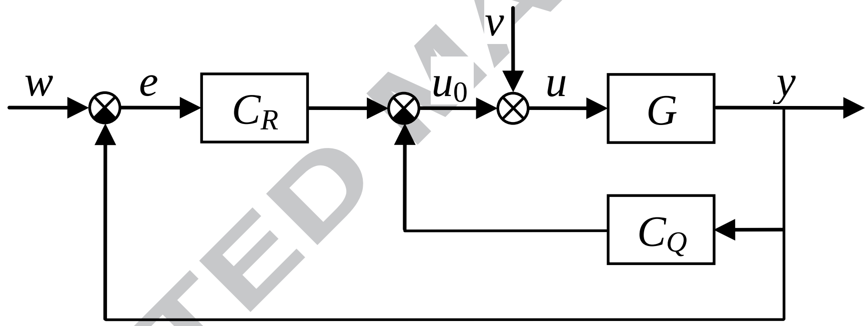

Fig. 6 – Control system with two feedback controllers

The blocks C_R and C_Q represent two controllers and G stands for a controlled plant. The symbols of the signals have the following meaning: w – reference signal, e – tracking (control) error, u_0 – difference of controllers' outputs, u – control signal, y – controlled signal (output), v – load disturbance.

The final finite-dimensional controllers, which were designed in [58], [62] by means of the algebraic method, are:

$$
\begin{aligned}
& C_R(s) = \frac{47.2676 s^2 + 0.416887 s + 6.16545 \times 10^{-4}}{s^2} \\
& C_Q(s) = 24.6114
\end{aligned} \tag{15}
$$

The family of the closed-loop characteristic retarded quasi-polynomials of the control loop from Fig. 6 with two feedback controllers (15) and heat exchanger model (12) with uncertain parameters (14) is:

$$\begin{aligned} & p_{CL}(s, b_0, b_{0D}, \Theta_0, \Theta_b, a_2, a_1, a_0, a_{0D}, \Theta_a) = s^3 + a_2 s^2 + a_1 s + a_0 + \cdots \\ & + a_{0D} e^{-\Theta_a s} + b_0 e^{-\Theta_b s} \left(47.2676 s^2 + 0.416887 s + 24.612016545\right) + \cdots \\ & + b_{0D} e^{-(\Theta_0 + \Theta_b) s} \left(47.2676 s^2 + 0.416887 s + 24.612016545\right) \end{aligned} \tag{16}$$

where the uncertain parameters can vary within the intervals from (14).

The value sets of the family (16) for frequencies from 0 to 2 with the step 0.02 are shown in Fig. 7. In this case, 7 out of 9 uncertain parameters are sampled by 3 equidistant values and the remaining 2 parameters are sampled by 15 values. It means that the value set consist of $3^7 \cdot 15^2 = 492075$ points at each frequency. The Fig. 8 represents a zoomed version of the sample plot as in Fig. 7. The family (16) contains a stable member [60] and the zero point is excluded from the value set, which means the family is robustly stable. In other words, two feedback controllers (15) robustly stabilize the anisochronic model (12) of a heating-cooling system with a heat exchanger for all possible variations of the uncertain parameters (14).

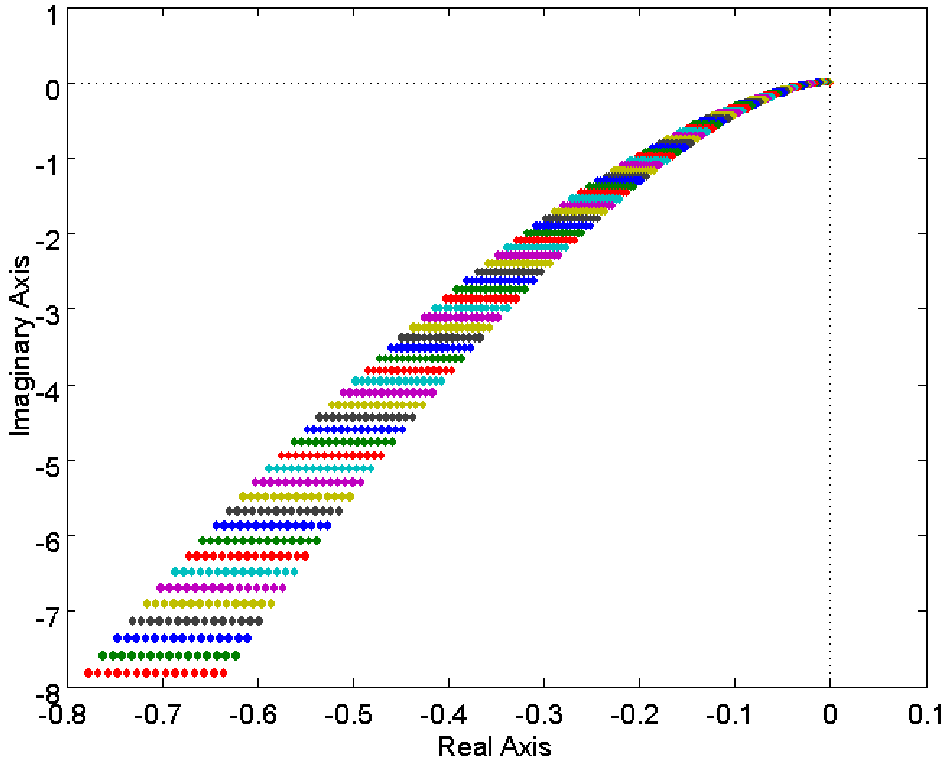

Fig. 7 – The value sets of the family (16) with parameters (14) – the full view

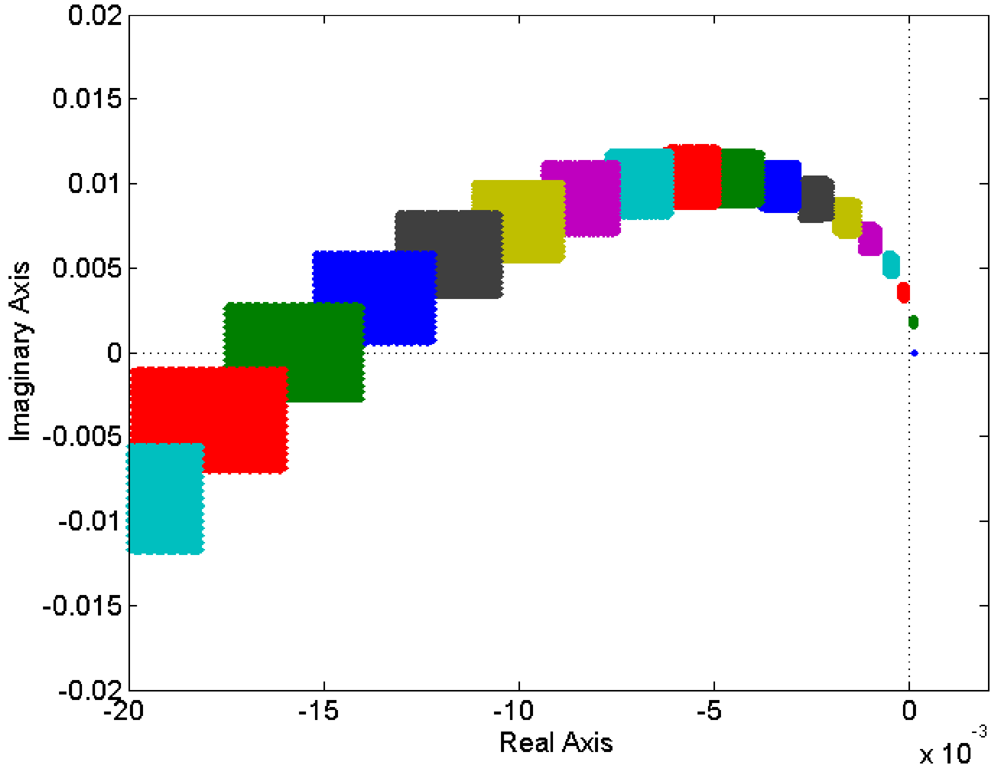

Fig. 8 – The value sets of the family (16) with parameters (14) – a zoomed view

4. Conclusion

This example-oriented paper was focused on the analysis of robust stability for thermal control systems with a fixed integer or fractional order controller (or two feedback controllers) and integer or fractional order controlled plant with time-delay(s) and uncertain parameters. The applied graphical technique is based on a universal principle that combines the numerical calculations of the value sets and the application of the zero exclusion condition. A trio of analysis examples inspired by control of the real thermal processes [50], [56], [58] is presented for the purpose of demonstrating the relative simplicity and ease of use but also the efficiency of the method. Moreover, to the best of authors' knowledge, the graphical analysis of robust stability for the loop with the anisochronic model with internal delays and parametric uncertainty, presented in the Subsection 3.3, was not previously published to date.

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Highlights:

- A method for robust stability analysis of thermal control systems is presented.
- The graphical method is applicable for various systems with parametric uncertainty.
- The technique is extremely universal and relatively easy to use.
- Even very complicated systems (with non-existing analytical tools) can be analyzed.
- The robust stability is investigated with the necessary and sufficient condition.