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REGIONS OF STABILITY FOR PID CONTROLLERS

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Abstract: *This contribution deals with calculating and plotting the stability regions for Proportional-Integral-Derivative (PID) controllers. The region of possible placement of the PID controller parameters which assure feedback stabilization of a controlled system is obtained by means of the Tan's method. This technique is based on plotting the stability boundary locus in various planes of the P-I-D space. Possible application is shown on an illustrative example where a third order plant is stabilized*

Key words: *PID Controllers, stability regions, stabilization*

1. INTRODUCTION

An overwhelming majority of nowadays practical industrial control applications employs PI(D) compensators (Åström & Hägglund, 1995). For that reason, the investigation on appropriate design methods for PID controllers is still very topical (O'Dwyer, 2003), especially if these controllers are able to cope with plants under various nonlinearities, perturbations or time-variability. However, the primary and essential requirement consists in ensurance of closed-loop stability.

There are a number of methods for computation of stabilizing PI(D) controllers in literature – e.g. calculation technique presented in (Söylemez et al., 2003), Tan's stability boundary locus approach from (Tan & Kaya, 2003; Tan et al., 2006) or method based on Kronecker summation published in (Fang et al., 2009). Application of the last two stabilization techniques, even from the robust stability viewpoint, has been already analyzed e.g. in (Matušů, 2008; Matušů & Prokop, 2010; Matušů, 2011a; Matušů, 2011b);

The contribution presents various approaches to plotting the 3-D stability regions in P-I-D space on the basis of Tan's stability boundary locus method (Tan et al., 2006). It partially draws upon the previously published works, especially (Matušů, 2011a) and (Matušů, 2011b). The applicability of described methodology is demonstrated on an illustrative example – stabilization of a third order plant.

2. CALCULATION OF STABILITY REGIONS

Assume a conventional feedback control loop with a controlled plant:

$$G(s) = \frac{b(s)}{a(s)} \tag{1}$$

and with an ideal PID controller:

$$C(s) = k_P + \frac{k_I}{s} + k_D s = \frac{k_P s + k_I + k_D s^2}{s} \tag{2}$$

A possible approach for computation of all possible parameters of the PID controller which guarantee closed-loop stability has been published in (Tan et al., 2006). Besides, its simplified version suitable for PI controllers with additional embellishment also for interval controlled systems can be found e.g. in (Tan & Kaya, 2003). The method is based on plotting the stability boundary locus. As the first step, the substitution $s = j\omega$ in (1) and consequent decomposition of the numerator and denominator into their even and odd parts results in:

$$G(j\omega) = \frac{b_E(-\omega^2) + j\omega b_O(-\omega^2)}{a_E(-\omega^2) + j\omega a_O(-\omega^2)} \tag{3}$$

Subsequently, computation of closed-loop characteristic polynomial and setting the real and imaginary parts to zero lead to equations for proportional and integral gains (Tan & Kaya, 2003; Tan et al., 2006; Matušů, 2011a; Matušů, 2011b):

$$\begin{aligned} k_P(\omega, k_D) &= \frac{P_5(\omega)P_4(\omega) - P_6(\omega)P_2(\omega)}{P_1(\omega)P_4(\omega) - P_2(\omega)P_3(\omega)} \\ k_I(\omega, k_D) &= \frac{P_6(\omega)P_1(\omega) - P_5(\omega)P_3(\omega)}{P_1(\omega)P_4(\omega) - P_2(\omega)P_3(\omega)} \end{aligned} \tag{4}$$

where

$$\begin{aligned} P_1(\omega) &= -\omega^2 B_O(-\omega^2) \\ P_2(\omega) &= B_E(-\omega^2) \\ P_3(\omega) &= \omega B_E(-\omega^2) \\ P_4(\omega) &= \omega B_O(-\omega^2) \\ P_5(\omega) &= \omega^2 A_O(-\omega^2) + \omega^2 B_E(-\omega^2) k_D \\ P_6(\omega) &= -\omega A_E(-\omega^2) + \omega^3 B_O(-\omega^2) k_D \end{aligned} \tag{5}$$

Simultaneous solution of relations (4) and plotting the obtained values into the (k_P, k_I, k_D) space define the stability boundary locus.

Note that the parameters k_P and k_I depend on derivative constant k_D, which is practically considered to be chosen and fixed for one set of calculations. In other words, k_D is preset and corresponding set of boundary parameters k_P, k_I is consequently calculated. The obtained curve splits the (k_P, k_I) plane into the stable and unstable regions. The decision on the stabilizing/unstabilizing regions can be done via a test point within each area. Then, the final stability region(s) can be successively plotted through the "(k_P, k_I) sections" in the (k_P, k_I, k_D) space by sampling k_D.

Analogically, the stability boundary locus can be obtained by means of (k_P, k_D) pairs for the sampled values of k_I. However, the third option of getting the stability boundary – in the (k_I, k_D) plane for a fixed value of k_P – is not so straightforward because of division by zero. Nevertheless, the technique presented in (Tan et al., 2006), which concurs with linear programming based approach from (Ho et al., 1997), can be utilized.

3. ILLUSTRATIVE EXAMPLE

Consider a third order controlled plant which has been already used in (Matušů & Prokop, 2010) and which is given by transfer function:

$$G(s) = \frac{5}{s^3 + 2s^2 + 3s + 4} \tag{6}$$

It means that the even and odd parts from the transfer function (3) are:

$$\begin{aligned} B_E(-\omega^2) &= 5 \\ B_O(-\omega^2) &= 0 \\ A_E(-\omega^2) &= 2(-\omega^2) + 4 \\ A_O(-\omega^2) &= -\omega^2 + 3 \end{aligned} \tag{7}$$

First, the stability regions were calculated according to equations (4) and (5) (and bottomed by lines $k_I = 0$) for 41 equally spaced values of k_D from 0 to 1 and visualized in fig. 1. As can be verified, all the variations of PID controller parameters which are located inside the shape defined by stability regions from fig. 1 guarantee the feedback stabilization of the plant (6).

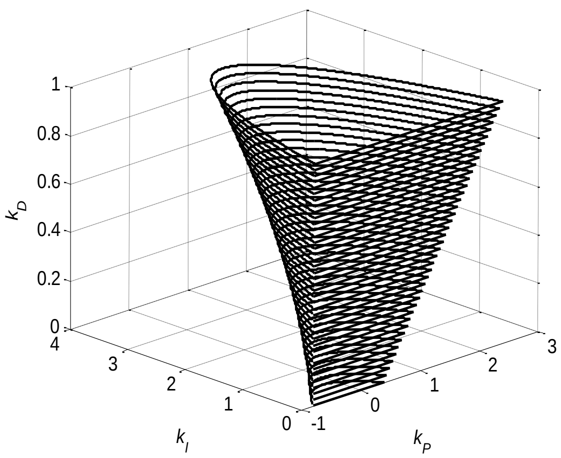

Fig. 1. Regions of stability for plant (6) and for $k_D \in \langle 0,1 \rangle$

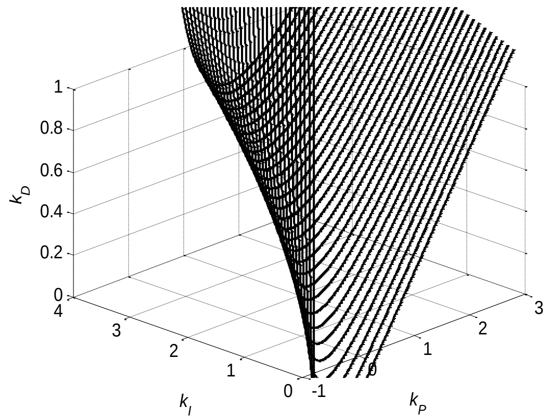

Fig. 2. Regions of stability for plant (6) and for $k_I \in \langle 0,3 \rangle$

Then, the stabilizing regions were computed alternatively by cutting the sections in (k_P, k_D) plane for 31 equally spaced values of k_I from 0 to 3. The obtained results are shown in fig. 2. The final 3-D stability regions from both fig. 1 and fig. 2 cover the same area.

4. CONCLUSION

The contribution was mainly intended to present various approaches to computing and consequent plotting the 3-D stability regions in P-I-D space. The applied idea has been based on Tan's stability boundary locus technique (Tan et al., 2006). In the illustrative example, the final stability region for PID controller and the third order controlled plant was obtained first using "(k_P, k_I) sections" for fixed samples of k_D and then by means of "(k_P, k_D) sections" for sampled values of k_I .

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