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DELTA MODELS IN ADAPTIVE CONTROL OF NONLINEAR PROCESS

BABIK, Z[denek]

Abstract: *The application of delta models in adaptive control of nonlinear process is described in this paper. Pole placement method is used for design a t-continuous controller and structure with two feedback controllers (TFC) is supposed. Equations which are used for design are presented too. This method has been tested on a real model DR300-AMIRA – speed servomechanism.*

Key words: *Nonlinear process, delta model, pole placement, TFC – Two Feedback Controllers*

1. INTRODUCTION

Control of a nonlinear or t-variant process using fixed set PID controllers causes many negative effects which usually impair the quality of regulation and reduce the lifetime of a controlled facility. We describe one way how to reduce these problems in this paper. The main aim is the approximation of a time-continuous model of controlled process by a discrete δ-model and continually identifying of its parameters.

2. DELTA MODELS

Delta models represented a link between t-continuous S-models and discrete Z-models. The parameters of this discrete models are convergented to the parameters of t-continuous models for a short sample time period (with regard to a dynamism of process).

When we are identifying the parameters we will be look for the δ-model whose structure will be same as the structure of t-continuous model. We suppose that the period of identification process is much shorter then dynamism of process. The parameters of the δ-model are much closed to the parameters of t-continuous model of controlled process.

EXTERNAL LINEAR δ-MODEL OF CONTROLLED PROCESS

T-continuous external linear model (ELM) is choosen on the basis of fundamental knowledge of dynamics of nonlinear control process. This model is described by a difference equation in time domain.

$$a(\sigma)y(t) = b(\sigma)u(t) \tag{1}$$

and in a complex domain by a transfer function:

$$G(s) = \frac{b(s)}{a(s)} \tag{2}$$

with conditions of rightness $\deg b \leq \deg a$.

DELTA MODEL OF PROCESS

We establish the δ-operator which is defined like:

$$\delta = \frac{q-1}{T_0} \tag{3}$$

where: q presents a shift operator and T_0 a sample time.

When the sample time period is shorted, then the δ-operator approximates the derivative operator σ so that it applies:

$$\lim_{T_0 \to 0} \delta = \sigma \tag{4}$$

and δ-model

$$a'(\delta)y(t') = b'(\delta)u(t') \tag{5}$$

approximates the t-continuous model. In (5) operator t' is a discrete time and a', b' are δ–polynomials.

3. SPEED SERVOMECHANISM MODEL

T-continuous model of second order has been choosen as:

$$\ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) = b_0 u(t) \tag{6}$$

Delta model which is corresponding with (6) has a form:

$$\delta^2 y(t') + a'_1 \delta y(t') + a'_0 y(t') = b'_0 u(t') \tag{7}$$

Regression vector has a form:

$$\Phi_\delta^T(k-1) = [-\varphi_y(k-2) \quad -\varphi_y(k-1) \quad \varphi_u(k-2)] \tag{8}$$

where:

$$\varphi_y(k-2) = y(k-2) \tag{9}$$

$$\varphi_y(k-1) = \frac{y(k-1) - y(k-2)}{T_0} \tag{10}$$

$$\varphi_u(k-2) = u(k-2) \tag{11}$$

Vector of delta model parameters

$$\Theta_\delta^T(k) = [a'_0 \quad a'_1 \quad b'_0] \tag{12}$$

is recursively estimated from the equation:

$$\varphi_y(k) = \Theta_\delta^T(k)\Phi_\delta(k-1) + \varepsilon(k) \tag{13}$$

where:

$$\varphi_y(\mathrm{k}) = \frac{y(k) - 2y(k-1) + y(k-2)}{T_0^2} \tag{14}$$

For the identification of parameters has been used recursive identification algorithm with directional forgetting. More information can be found for example in (Bobál, 2008).

4. TWO FEEDBACK CONTROLLERS (TFC)

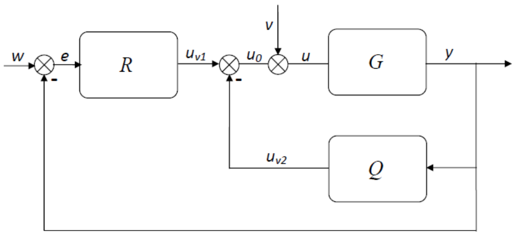

Fig. 1. Control loop with TFC controller

Transfer in a control loop

G – linear input-output model of control process
Q, *R* – feedback controllers

Signals in a control loop

w – reference signal
e – control error
u_{v1}, u_{v2} – control action
v – system intput disturbance
u – process-input-control action
y – output signal

Transfer functions of both feedback controllers are considered in the form:

$$Q(s) = \frac{\tilde{q}(s)}{\tilde{p}(s)} \quad \mathrm{R}(s) = \frac{r(s)}{\tilde{p}(s)} \tag{15}$$

where $\tilde{q}(s)$, $\tilde{p}(s)$ a $r(s)$ are relative prime polynomials which are expressed in terms of complex variable s.

In this case, both input signals w a v (reference signal and disturbance) are considered only as a step function with the images:

$$W(s) = \frac{w_0}{s} \quad V(s) = \frac{v_0}{s} \tag{16}$$

For the output signal and for the disturbance can be derived (the argument s is omitted for clarity):

$$Y(s) = \frac{b}{d}[rW(s) + \tilde{p}V(s)] \tag{17}$$

$$E(s) = \frac{1}{d}[(a\tilde{p} + b\tilde{q})W(s) - b\tilde{p}V(s)] \tag{18}$$

where:

$$d(s) = a(s)\tilde{p}(s) + b(s)\cdot(r(s) + \tilde{q}(s)) \tag{19}$$

where $d(s)$ is a characteristic polynomial with roots that are represented the poles of the closed control loop.

Now we choose the polynomial $t(s)$ which meets a condition:

$$t(s) = \mathrm{r}(s) + \tilde{\mathrm{q}}(s) \tag{20}$$

Polynomial $t(s)$ we substitute into (19). The stability condition will be satisfied if the polynomials $\tilde{\mathrm{p}}(s)$ and $t(s)$ are given by a solution of a Diophantine equation:

$$a(s)\tilde{p}(s) + b(s)\cdot t(s) = d(s) \tag{21}$$

with a stable polynomial $d(s)$ on the right side.

The asymptotic tracking of a reference signal and the elimination of a disturbance will be guaranteed for a step input signals if the conditions $\mathrm{a}\tilde{\mathrm{p}} + \mathrm{b}\tilde{\mathrm{q}}$ and $\tilde{\mathrm{p}}$ contain s.

These conditions will be fulfilled if the polynomials $\tilde{\mathrm{p}}$ and $\tilde{\mathrm{q}}$ are in the form:

$$\tilde{p}(s) = sp(s) \tag{22}$$

$$\tilde{q}(s) = sq(s) \tag{23}$$

The transfer function of controllers we can consider in the form:

$$Q(s) = \frac{q(s)}{p(s)} \quad R(s) = \frac{r(s)}{s\cdot p(s)} \tag{24}$$

The stability of controllers is garanted by a stable polynomial $p(s)$ in a denominator of (24).

The degree of polynomials $q(s)$ and $r(s)$ must satisfy the following inequality to meet the conditions of inner purity control system:

$$\deg q \le \deg p \tag{25}$$

$$\deg r \le \deg p + 1 \tag{26}$$

The polynomial $t(s)$ we rewrite to the form:

$$t(s) = r(s) + s\cdot q(s) \tag{27}$$

When we considering the solvability of (22) and conditions (26) and (27), the degree of polynomials may be identified as:

$$\deg t = \deg r = \deg a \tag{28}$$

$$\deg q \ge \deg a - 1 \tag{29}$$

$$\deg p \ge \deg a - 1 \tag{30}$$

$$\deg d \ge 2\deg a \tag{31}$$

If $\deg a = n$ then the polynomials t, r and q are in the form:

$$t(s) = \sum_{i=0}^{n} t_i s^i \quad r(s) = \sum_{i=0}^{n} r_i s^i \quad q(s) = \sum_{i=1}^{n} q_i s^{i-1} \tag{32}$$

where the coefficients r_i, q_i and t_i are considered the conditions:

$$r_0 = t_0 \tag{33}$$

$$r_i + q_i = t_i \text{ pro } i=1,\dots,n \tag{34}$$

Unknown coefficients r_i and q_i can be calculated by using a optional factors $\beta_i \in \langle 0{,}1\rangle$:

$$r_i = \beta_i \cdot t_i \tag{35}$$

$$q_i = (1 - \beta_i)\cdot t_i \text{ pro } i=1,\dots,n \tag{36}$$

The coefficients β_i distribute a weight between numerators of transfer functions Q and R. It is true, if the value of β_i is increased, the response to a step change is accelerating.

Notice: If $\beta_i = 1$ for all i, the TFC engagement corresponding with 1DOF system control configuration and if $\beta_i = 0$ for all i and setpoint and disturbance are a step functions, it corresponding with 2DOF system control configuration.

5. REAL MEASUREMENTS

The transfer functions of controllers have a form:

$$Q(s) = \frac{U_{v2}(s)}{Y(s)} = \frac{q_2 s + q_1}{p_1 s + p_0} \tag{37}$$

$$R(s) = \frac{U_{v1}(s)}{E(s)} = \frac{r_2 s^2 + r_1 s + r_0}{s(p_1 s + p_0)} \tag{38}$$

Characteristic polynomial: $d(s) = n(s)(s + 0.9)^2$ where the polynomial $n(s)$ is a product of spectral factorization of the polynomial $a(s)$.

Controller parameters: $\beta = 0$

Sample time period: $T_0 = 0.025s$

Duration of the control: $t = 300s$

Identification process parameters: $C_{ii} = 1000, \varphi(0) = 1,$ $\lambda(0) = 0.001, \upsilon(0) = 10^{-6},\ \varrho = 0.99$

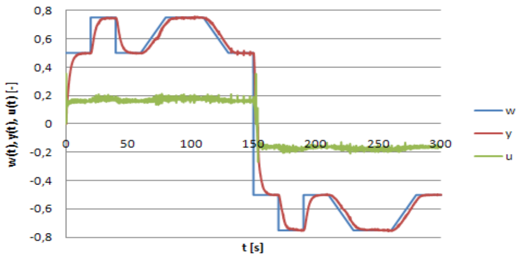

Fig. 2. Control performance $\beta = 0$

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