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Control of Interval Systems Using 2DOF Configuration

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Abstract

This contribution is focused on continuous-time control of interval systems by means of two-degree-of-freedom (2DOF) configuration. The controller design utilizes algebraic techniques in the ring of proper and (Hurwitz-)stable rational functions (R_{PS}). Robust stability of resulting 2DOF loops is analyzed graphically, namely with the assistance of the value set concept and the zero exclusion condition. In the presented illustrative example, a third order interval plant is robustly stabilized.
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Keywords: 2DOF control structure; algebraic approach; parametric uncertainty; value set concept; zero exclusion condition

1. Introduction

The control systems with two-degree-of-freedom (2DOF) structure are loops with separated feedback and feedforward parts. In comparison with classical one-degree-of-freedom (1DOF) configurations they have considerable advantages [1]-[3].

This contribution focuses on possible application of continuous-time 2DOF controllers to plants affected by parametric uncertainty. Such kind of controlled systems is supposed to have known structure (known order) but the parameters themselves can lie within some bounds. Usually, they are bounded by real intervals with minimal and maximal possible values. For the sake of this paper, the 2DOF controllers are designed with the assistance an algebraic approach, more specifically by solution of Diophantine equations in the ring of proper and stable rational functions [4]-[10]. Consequent robust stability analysis utilizes the combination of the value set concept with the zero exclusion condition [11]. Even the approach is applicable for general systems with parametric uncertainty, the illustrative example deals with typical case of interval system. More specifically, a third order interval plant with

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perturbed parameters of the size ±30% is robustly stabilized. The similar ideas as in this work have been already published in the paper [12] and its extended version [13].

2. Control synthesis

The Fig. 1 shows the closed-loop control system with 2DOF structure. The transfer functions $C_b(s)$, $C_f(s)$, and $G(s)$ represent feedback part of the controller, feedforward part of the controller, and controlled plant, respectively, and the signals $w(s)$, $n(s)$, and $v(s)$ describe reference, load disturbance, and disturbance signal.

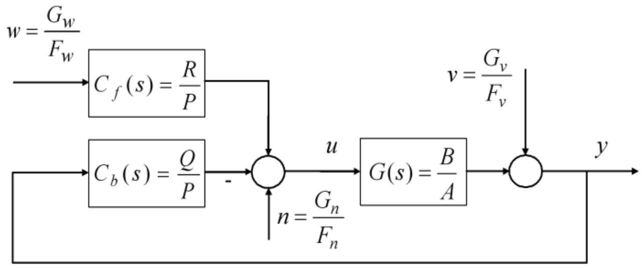

Fig. 1. 2DOF control system.

The basic ideas of the control design method originate from the works of Vidyasagar [4] and Kučera [5]. The technique assumes the description of linear systems in Fig. 1 with the assistance of the ring of proper and stable rational functions (R_{PS}). The conversion from the ring of polynomials to R_{PS} can be accomplished in accordance with:

$$G(s) = \frac{b(s)}{a(s)} = \frac{b(s)}{(s+m)^{\max(\deg a, \deg b)}} \Bigg/ \frac{a(s)}{(s+m)^{\max(\deg a, \deg b)}} = \frac{B(s)}{A(s)} \tag{1}$$

The multiple root $m>0$ in denominator polynomials can be later used as a controller tuning parameter.

The assumption of no disturbances in the control system and subsequent algebraic analysis (see e.g. [9], [10]) lead to the first Diophantine equation:

$$A(s)P(s) + B(s)Q(s) = 1 \tag{2}$$

with a general solution $P(s) = P_0(s) + B(s)T(s)$, $Q(s) = Q_0(s) - A(s)T(s)$, where $T(s)$ is an arbitrary member of R_{PS} and the pair $P_0(s)$, $Q_0(s)$ represents the particular solution of (2). By this principle, which is known as Youla-Kučera parameterization, one can express all stabilizing feedback controllers. Since the feedback part of the controller is responsible not only for stabilization but also for possible disturbance rejection, the convenient controller from the set of all stabilizing ones can be chosen on the basis of divisibility conditions.

Then, the requirement of the reference tracking is formulated via the second Diophantine equation:

$$F_w(s)Z(s) + B(s)R(s) = 1 \tag{3}$$

As this contribution provides only a brief outline of the applied synthesis, further elaboration with detailed insight into the technique and also derivation of specific tuning rules can be found e.g. in [6] – [10].

3. Robust stability analysis for systems with parametric uncertainty

Generally, systems with parametric uncertainty can be described by:

$$G(s,q)=\frac{b(s,q)}{a(s,q)} \tag{4}$$

where q is a vector of uncertain parameters (uncertainty) confined by some uncertainty bounding set. Commonly, the controlled systems with parametric uncertainty are for the sake of simplicity considered as the interval plants which parameters can vary independently on each other within given bounds. Such kind of plants can be described by transfer functions with lower and upper limits for parameters:

$$G(s,b,a)=\sum_{i=0}^{m}\left[b_i^-;b_i^+\right]s^i \Big/ \sum_{i=0}^{n}\left[a_i^-;a_i^+\right]s^i \tag{5}$$

Since the stability of linear systems can be investigated through the stability of its characteristic polynomials, the primary object of interest from the robust stability viewpoint is the uncertain continuous-time closed-loop characteristic polynomial with general structure:

$$p(s,q)=\sum_{i=0}^{n}\rho_i(q)s^i \tag{6}$$

where $\rho_i(q)$ are coefficient functions. Then, so-called family of closed-loop characteristic polynomials can be denoted as:

$$P=\{p(\cdot,q):q\in Q\} \tag{7}$$

The family of polynomials (7) is robustly stable if and only if $p(s,q)$ is stable for all $q\in Q$. The selection of convenient technique for investigation of robust stability depends mainly on the structure of uncertainty, i.e. on the complexity and mutual connection of coefficient functions, while the higher level of relation among coefficients means the more complicated robust stability analysis. However, there exists a very universal graphical approach applicable for many, even very complicated, cases. It is known as the value set concept in combination with the zero exclusion condition [11].

Suppose a family of polynomials (7). The value set at frequency $\omega\in\Re$ is given by [11]:

$$p(j\omega,Q)=\{p(j\omega,q):q\in Q\} \tag{8}$$

The zero exclusion condition for Hurwitz stability of family of continuous-time polynomials (7) says [11]: Assume invariant degree of polynomials in the family, pathwise connected uncertainty bounding set Q, continuous coefficient functions $\rho_i(q)$ for $i=0,1,2,\dots,n$ and at least one stable member $p(s,q^0)$. Then the family P is robustly stable if and only if the complex plane origin is excluded from the value set $p(j\omega,Q)$ at all frequencies $\omega\geq 0$, that is P is robustly stable if and only if:

$$0\notin p(j\omega,Q) \quad \forall\omega\geq 0 \tag{9}$$

More details can be found in [11] or other related literature.

4. Illustrative example

The goal is to design 2DOF controller for step-wise reference tracking and potentially also step-wise load disturbance rejection which is able to robustly stabilize given third order interval system:

$$G(s,b_i,a_i)=\frac{[0.7;1.3]s^2+[0.7;1.3]s+[0.7;1.3]}{[0.7;1.3]s^3+[0.7;1.3]s^2+[0.7;1.3]s+[0.7;1.3]} \tag{10}$$

The fixed nominal system with average values of the uncertain parameters used for controller design is:

$$G_N(s)=\frac{s^2+s+1}{s^3+s^2+s+1} \tag{11}$$

Application of the synthesis method outlined in Section 2 and choice of the tuning parameter m=0.9 lead to the 2DOF controller for step-wise reference tracking and step-wise load disturbance rejection:

$$\begin{aligned} C_b(s)&=\frac{4.2699s^3+6.1685s^2+2.43s+0.5314}{s^3+0.1301s^2+0.5815s} \\ C_f(s)&=\frac{0.729s^3+1.9683s^2+1.7715s+0.5314}{s^3+0.1301s^2+0.5815s} \end{aligned} \tag{12}$$

The family of closed-loop characteristic polynomials (for the circuit with plant (10) and controller (12)) has the sixth order and so-called affine linear structure of uncertainty. The value sets plotted in concordance with the principle described in Section 3 for the range of frequencies $\omega=0:0.01:5$ is shown in left side of Fig. 2. Then, its zoomed version for better view in the neighbourhood of the complex plane origin is depicted in the right side of Fig. 2. Those plots can be obtained simply through the Polynomial Toolbox for Matlab [14]. Since the family contains a stable member and the complex plane origin is excluded from the value sets, it is robustly stable.

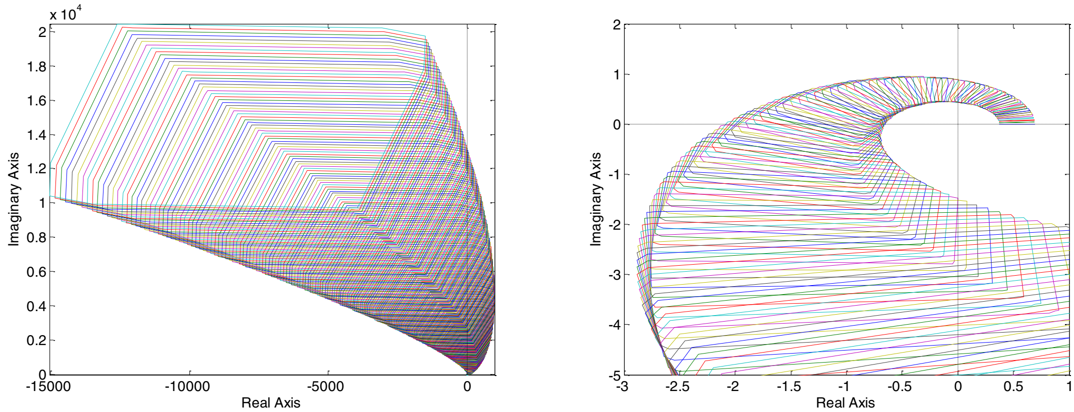

Fig. 2. The value sets for family of closed-loop characteristic polynomials under assumption of the plant (10) and controller (12) – full view (left) and zoomed version (right).

The control simulations also visually confirm robust stability of 2DOF loop. The Fig. 3 shows the output signals of the 128 “representative” systems from the interval family (10). For all interval parameters, minimal and maximal values are taken and thus 2 values and 7 parameters lead to $2^7=128$ systems for simulation. Moreover, the red curve represents the output signal for the nominal system (11). Furthermore, it was supposed the step-wise reference signal changing from 1 to 2 in one third of simulation time and the step-wise load disturbance -1 which influences the input to the controlled plant during the last third of simulation. Remind that this simulation (Fig. 3) itself is not the proof of robust stability, only visual confirmation of the fact.

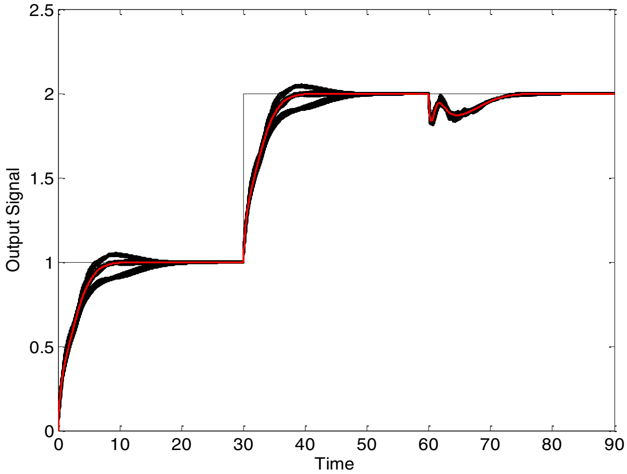

Fig. 3. Control of “representative” systems from interval plant (10) by 2DOF controller (12).

Conclusion

The contribution has been focused on R_{PS} design of 2DOF control laws and their application to interval systems. The synthesis method itself is followed by the graphical approach to robust stability analysis based on the value set concept and the zero exclusion condition. The main idea was illustrated through the simulation example.

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