Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 100 (2015) 249 – 258

Procedia Engineering

www.elsevier.com/locate/procedia

25th DAAAM International Symposium on Intelligent Manufacturing and Automation, DAAAM 2014

Modelling of Twin Rotor MIMO System

Petr Chalupa*, Jan Přikryl, Jakub Novák

Tomas BataUniversity in Zlin, nám. T.G.Masaryka 5555, Zlin 76001, Czech Republic

Abstract

The paper deals with modelling of a Twin Rotor MIMO System – a laboratory model constructed by Feedback Instruments Ltd. The system consists of a two rotors which resembles a simple helicopter. The non-stationary part of the model can rotate around two perpendicular axes to produce azimuth and elevation output of the system. These outputs are affected by speed of the rotors. Significant cross-coupling can be observed and the system behaviour is nonlinear. A model based on first–principle modelling was derived and is parameters were refined by identification based on real-time experiments. Resulting model was designed in MATLAB/Simulink environment and can serve for control design.

Peer-review under responsibility of DAAAM International Vienna

Keywords: Modelling; Simulink; Twin Rotor MIMO System; Nonlinear system

1. Introduction

Knowledge of a model a controlled plant is necessary for most of current control algorithms [1]. It is obvious that some information about controlled plant is required to allow design of a controller with satisfactory performance. A plant model can be also used to investigate properties and behaviour of the modelled plant without a risk of damage of violating technological constraints of the real plant. Two basic approaches of obtaining plant model exist: the black box approach and the first principles modelling (mathematical-physical analysis of the plant).

The black box approach to the modelling [2], [3] is based on analysis of input and output signals of the plant. The main advantage of this approach lies in the possibility of usage the same identification algorithm for wide set of various controlled plants [4]. Also, the knowledge of physical principle of controlled plant and solution of possibly complicated set of mathematical equation is not required. On the other hand, model obtained by black box approach

* Corresponding author. Tel.: +420 57603 5316; fax: +420 57603 5279.
E-mail address: chalupa@fai.utb.cz

Peer-review under responsibility of DAAAM International Vienna
doi:10.1016/j.proeng.2015.01.365

is generally valid only for signals it was calculated from [5]. For example, if just high frequency changes of input signals were used to obtain the model, this model need not be used for analysis of the plant behaviour in case of low frequency changes of input signals.

The first principle modelling provides general model valid for whole range of plant inputs and states. The model is created by analysing the modelled plant and combining physical laws [6]. On the other hand, there are usually many unknown constants and relations when performing analysis of a plant. Therefore, modelling by first principle modelling is suitable for simple controlled plants with small number of parameters. First principle modelling can be also used for obtaining basic information about controlled plant (rage of gain, rank of suitable sample time, etc.). Some simplifications must be used to obtain reasonable results in more complicated cases. These simplifications must relate with the purpose of the model. The first principle modelling is also referred to as white box modelling.

The paper uses combination of both methods. Basic relations between plant inputs and outputs are derived using first principles. The obtained model is further improved on the basis of measurements. This approach is known as grey box modelling [7]. The goal of the work was to obtain a mathematical model of the Twin Rotor MIMO System [8] and to design the model in MATLAB-Simulink environment. The Twin Rotor MIMO System was developed by Feedback Instruments Ltd. and serves as a real-time model of nonlinear multidimensional system. The major reason for creating the model of this laboratory equipment was usage of the model in control design process. A model, which represents the plant well, can considerably reduce testing time of different control approaches. Then only promising control strategies are applied to the real plant and verified.

The paper is organized as follows. Section 2 presents the modelled system – Twin Rotor MIMO System. A derivation of an initial ideal model using first principles modelling is carried out in Section 3. Enhancement of the model is presented in Section 4 and real-time experiments are described in Section 5. These experiments serve for the verification of the model by comparing its responses with the real-time plant.

The following notation is used throughout the paper:

Nomenclature

m_{mr}	The mass of the main DC-motor with main rotor
m_{ms}	The mass of the main shield
m_m	The mass of main part of the beam
l_m	The length of main part of the beam
m_{tr}	The mass of the tail motor with tail rotor
m_{ts}	The mass of the tail shield
m_t	The mass of the tail part of the beam
l_t	The length of tail part of the beam
m_{cb}	The mass of the counter-weight
m_b	The mass of the counter-weight beam
l_{cb}	The length of the counter-weight beam
g	Gravitational acceleration
M_v	Moment of forces in the vertical plane
J_v	Moments of inertia relative to the horizontal axis (i.e. in the vertical plane)
α_v	The pitch angle of the beam – elevation
ω_m	The angular velocity of the main rotor
$F_v(\omega_m)$	The dependence of the propulsive force on the angular velocity of the main rotor
α_h	The azimuth of the beam
k_v	A constant of a friction
r_{ms}	The radius of the main shield
r_{ts}	The radius of the tail shield
M_h	Moment of forces in the horizontal plane
J_h	Moments of inertia relative to the vertical axis (i.e. in the horizontal plane)
ω_t	The angular velocity of the tail rotor
$F_h(\omega_t)$	The dependence of the propulsive force on the angular velocity of the tail rotor
k_v	A friction constant
S_v	The angular momentum in vertical plane for the beam

S_h	The angular momentum in horizontal plane for the beam
J_{tr}	The moment of inertia in DC-motor – tail propeller subsystem
J_{mr}	The moment of inertia in DC-motor – main propeller subsystem
T_{mr}	The time constant of the main motor – propeller system
$P_v(u_{vv})$	The static nonlinearity of the main motor – propeller system
T_{tr}	The time constant of the tail motor – propeller system
$P_h(u_{hh})$	The static nonlinearity of the tail motor – propeller system
u_v	The control voltage of the maim motor
u_h	The control voltage of the tail motor

2. Twin Rotor MIMO System

A photograph of the Twin Rotor MIMO System is presented in Fig. 1. The system is used to demonstrate the principles of a non-linear MIMO system, with significant cross-coupling. Its behaviour resembles a helicopter but contrary to most flying helicopters the angle of attack of the rotors is fixed and the aerodynamic forces are controlled by varying the speeds of the motors. Significant cross-coupling is observed between the actions of the rotors, with each rotor influencing both angle positions [9]. Some aspects of the plant behaviour are similar to flying helicopters [10].

There are two propellers driven by DC-motors at both ends of a beam, which is pivoting on its base. The joint allows the beam to rotate in such a way that its ends move on spherical surfaces. There is a counter-weight fixed to the beam and it determines a stable equilibrium position. The controls of the system are the motors supply voltages. The measured signals are position of the beam in the space, i.e. two position angles [11].

Fig. 1. Twin Rotor MIMO System.

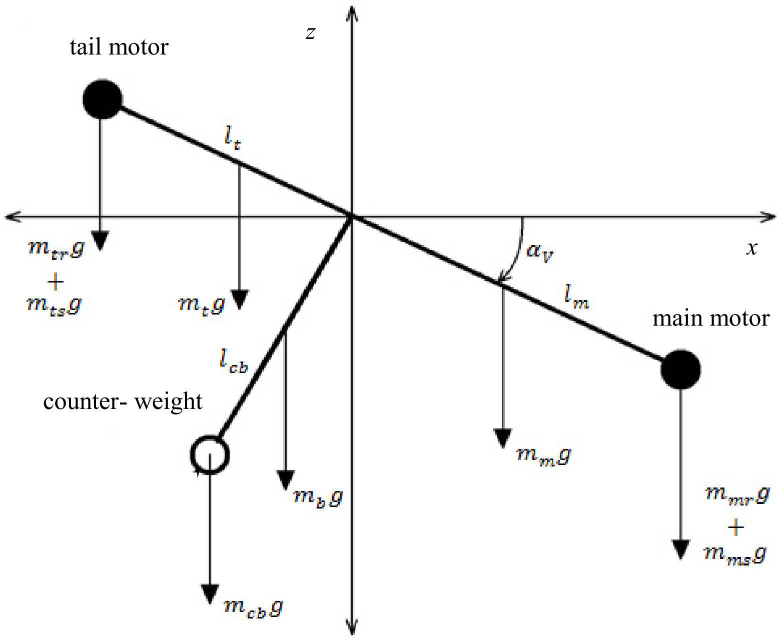

Fig. 2. Schematic front view of the Twin Rotor MIMO System with gravitation forces.

3. Initial model of the Twin Rotor MIMO System

A nonlinear model of the plant is derived in this section. The model is based on first principle modeling [11], [12], and [13]. There are the outputs of the plant: position angle in the vertical plane – elevation (i.e. angle with respect to horizontal axis) and position in the horizontal plane – azimuth (i.e. angle with respect to vertical axis). First vertical plane will be considered, and then horizontal angle will be focused on. A schematic front view of the free beam and connected parts of the Twin Rotor MIMO System is depicted in Fig. 2. The gravitation forces taking effect are presented as well.

3.1. Moments in the vertical plane

The derivation is based on Newton's second law of motion:

$$M_v = J_v \frac{d^2\alpha_v}{dt^2} \tag{1}$$

where M_v is a sum of components of moment of forces and J_v is a sum of moments of inertia relative to horizontal axis of individual parts of the plant:

$$M_v = \sum M_{vi} \tag{2}$$

$$J_v = \sum J_{vi} \tag{3}$$

Individual moments of forces can be expressed by analyzing Fig. 2. It is possible to derive the following equations. The moment caused by gravitational forces:

$$M_{v1} = g[(A - B)\cos\alpha_v - C\sin\alpha_v] \quad (4)$$

where

$$A = \left(\frac{m_t}{2} + m_{tr} + m_{ts}\right) l_t, \quad B = \left(\frac{m_m}{2} + m_{mr} + m_{ms}\right) l_m, \quad C = \left(\frac{m_b}{2} + m_{cb}\right) l_{cb} \quad (5)$$

The moments of propulsive forces applied to the beam can be expressed as

$$M_{v2} = l_m F_v(\omega_m) \quad (6)$$

The moment of centrifugal forces corresponding to the motion of the beam around the vertical axis can be express as follows:

$$M_{v3} = -\Omega_h^2 (A + B + C)\sin\alpha_v \cos\alpha_v, \quad \Omega_h = \frac{d\alpha_h}{dt} \quad (7)$$

The last component of M_v corresponds to the moment of friction depending on the angular velocity of the beam around the horizontal axis.

$$M_{v4} = -\Omega_v k_v, \quad \Omega_v = \frac{d\alpha_v}{dt} \quad (8)$$

Moments of inertia corresponding to individual parts of the plant can be expressed as follows:

$$J_{v1} = m_{mr} l_m^2, \quad J_{v2} = m_m \frac{l_m^2}{3}, \quad J_{v3} = m_{cb} l_{cb}^2, \quad J_{v4} = m_b \frac{l_{cb}^2}{3},$$

$$J_{v5} = m_{tr} l_t^2, \; J_{v6} = m_t \frac{l_t^2}{3}, \quad J_{v7} = \frac{m_{ms}}{2} r_{ms}^2 + m_{ms} l_m^2, \quad J_{v8} = m_{ts} r_{ts}^2 + m_{ts} l_t^2 \quad (9)$$

3.2. Moments in the horizontal plane

The moments present in the horizontal plane can be derived in the similar way as the moments in the horizontal plane. The moments of propulsive forces can be expressed as:

$$M_{h1} = l_t F_h(\omega_t)\cos\alpha_v \quad (10)$$

And the moment of friction is defined by the following equation:

$$M_{h2} = -\Omega_h k_h \quad (11)$$

The moment of inertia relative to the vertical axis is dependent on pitch position of the beam and can be expressed in the compact form:

$$J_h = D\sin^2\alpha_v + E\cos^2\alpha_v + F \quad (12)$$

where

$$D = \left(\frac{m_b}{3} + m_{cb}\right) l_{cb}^2, \quad E = \left(\frac{m_m}{3} + m_{mr} + m_{ms}\right) l_m^2 + \left(\frac{m_t}{3} + m_{tr} + m_{ts}\right) l_t^2, \quad F = m_{ms} r_{ms}^2 + \frac{m_{ts}}{2} r_{ts}^2 \quad (13)$$

3.3. State equations

By combining equations (1) – (13), it is possible to derive state equations of the whole system:

$$\frac{dS_v}{dt} = \frac{g[(A-B)\cos\alpha_v - C\sin\alpha_v] + l_m F_v(\omega_m) - \Omega_h^2 (A+B+C)\sin 2\alpha_v - \Omega_v k_v}{J_v} \tag{14}$$

$$\frac{dS_\mathrm{h}}{dt} = \frac{l_t F_h(\omega_t)\cos\alpha_v - \Omega_h k_h}{J_h} = \frac{l_t F_h(\omega_t)\cos\alpha_v - \Omega_h k_h}{D\sin^2\alpha_v + E\cos^2\alpha_v + F} \tag{15}$$

$$\Omega_v = S_v + \frac{J_{tr}\omega_t}{J_v} \tag{16}$$

$$\Omega_h = S_h + \frac{J_{mr}\omega_m \sin\alpha_v}{J_v} \tag{17}$$

These equations describe dependence of output angles (elevation α_v and azimuth α_h) on rotations of the main and the tail motors – ω_m and ω_t respectively. The motors are controlled by control voltage according to the following combinations of linear dynamics and static non-linearity:

$$\frac{du_{vv}}{dt} = \frac{1}{T_{mr}}(-u_{vv} + u_v), \quad \omega_m = P_v(u_{vv}) \tag{18}$$

$$\frac{du_{hh}}{dt} = \frac{1}{T_{mr}}(-u_{hh} + u_h), \quad \omega_t = P_h(u_{hh}) \tag{19}$$

4. Enhanced model

Documentation [11] provides parameters and relations of the Twin Rotor MIMO System which are present in equations in section 3. However real-time experiments showed that these parameters and equations should be refined or revised.

This section is focused on modification of the initial model presented in Section 3 and parameters given in [11] in order to obtain better correspondence of the mathematical model and real time system. As the Section 3 was related to white box modeling this Section represents grey box modeling.

4.1. Refinement of the dimensions

The dimensions of the modeled Twin Rotor MIMO System were measured to refine constants given in documentation [11]. Table 1 summarizes the differences.

Table 1. Dimensions of the Twin Rotor MIMO System.

Parameter	Documentation [m]	Measured dimension [m]
l_t	0,25	0,28
l_m	0,24	0,25
l_{cb}	0,13	0,24
r_{ms}	0,155	0,155
r_{ts}	0,10	0,10

It can be seen that especially the length of the counter-weight beam is different from the value given in documentation.

4.2. Nonlinear static functions

The nonlinear functions $F_v()$, $F_h()$, $P_v()$ and $P_h()$ are used in equations (6), (10), (18) and (19) respectively. These functions have to be determined to design the final model. A phenomenological approach was used for their

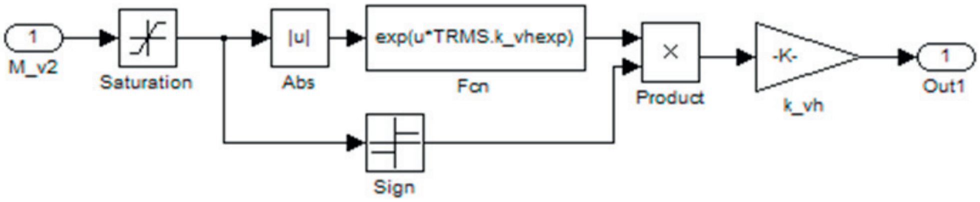

Fig. 3. Model of influence of main motor to the azimuth.

identification. A polynomial approximation was used without deep study of the physical fundamentals of the relation. Following relation are used in the final model:

$$P_v(u_{vv}) = 90{,}99u_{vv}^6 + 599{,}73u_{vv}^5 - 129{,}26u_{vv}^4 - 1238{,}64u_{vv}^3 + 63{,}45u_{vv}^2 + 1283{,}41u_{vv} \tag{20}$$

$$F_v(\omega_m) = 3{,}187 \cdot 10^{-12}\omega_m^5 - 4{,}096 \cdot 10^{-9}\omega_m^4 + 1{,}385 \cdot 10^{-6}\omega_m^3 + 1{,}234 \cdot 10^{-3}\omega_m^2 + 0{,}799\omega_m \tag{21}$$

$$P_h(u_{hh}) = 2020u_{hh}^5 - 194{,}69u_{hh}^4 - 4283{,}15u_{hh}^3 + 262{,}27u_{hh}^2 + 3796{,}83u_{hh} \tag{22}$$

$$F_h(\omega_t) = 9{,}496 \cdot 10^{-13}\omega_t^5 - 9{,}844 \cdot 10^{-10}\omega_t^4 + 2{,}785 \cdot 10^{-7}\omega_t^3 + 1{,}730 \cdot 10^{-4}\omega_t^2 + 0{,}729\omega_t \tag{23}$$

4.3. Cross coupling transfers

The cross coupling can be observed in the Twin Rotor MIMO System. The rotation of the tail motor slightly affects elevation angle while main motor strongly affects not only elevation but also azimuth. The influence of tail motor to elevation was modeled as linear function of tail rotor rotations.

The dependence of azimuth on rotations of the main motor is more complicated to model. An exponential function of the M_{v2} moment was used to cope with this problem. A Simulink scheme of this relation is presented in Fig. 3.

4.4. Cableways

A cableway between the fixed base of the Twin Rotor MIMO System and its beam plays a significant role especially in case of low rotation speed of the tail motor. Due to the cable way the system does not behave as an integrative but proportional behaviour can be observed. The effect of the cableway is modelled as a nonlinear function of the azimuth.

5. Real time experiments

5.1. Static characteristics

Static characteristic of the elevation was measured for increasing and decreasing course of control voltage of main rotor. The tail motor was not rotating during the experiment. The output is measured in Matlab Units (MU), i.e. direct output of the incremental sensor. Resulting course is presented in Fig. 4. It can be observed that the direction of input voltage change (i.e. increasing or decreasing) does not play significant role. The static characteristic can be considered a piece-wise linear. When the direction of the propeller rotation changes, which corresponds to the change of sign of control voltage, the gain of the system changes.

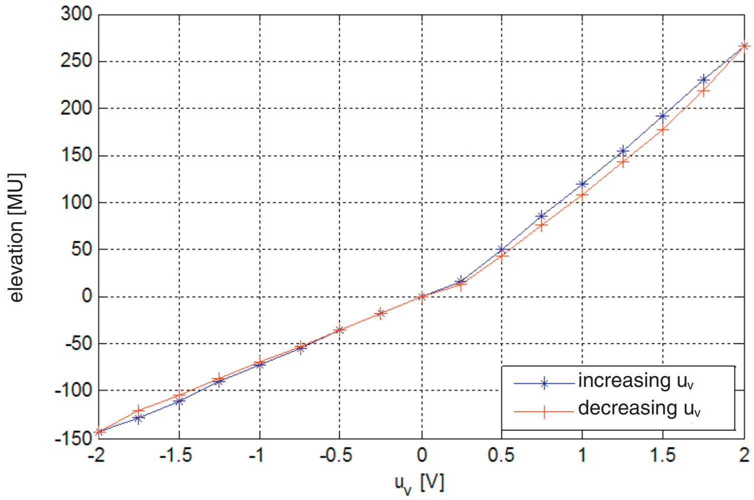

Fig. 4. Static characteristics of the main motor – elevation system.

5.2. Step responses

Several step responses were measured to compare behaviour of the model, which was designed in the MATLAB / Simulink environment, and the real time system. As the static characteristics of both outputs were measures as a series of the step changes of input, the linear model can be identified from these step responses. The linear model of the main motor – elevation subsystem was identified using *fminsearch* MATLAB function:

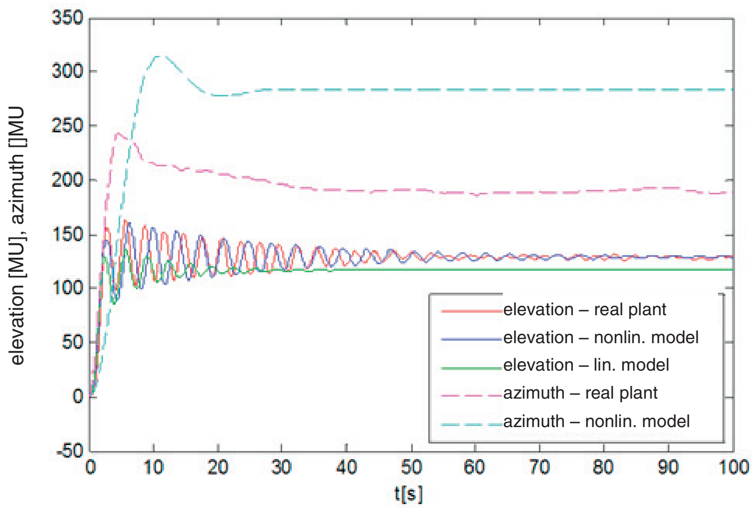

Fig. 5. Comparison of step responses for change of control voltage of main motor from 0V to +1V.

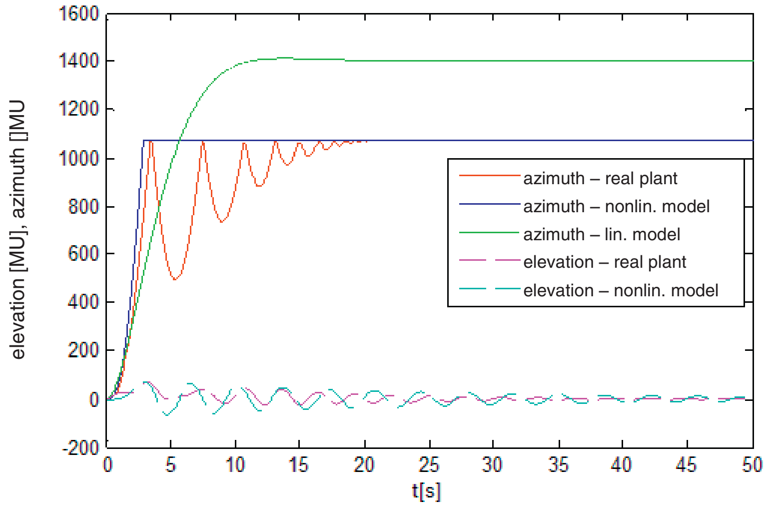

Fig. 6. Comparison of step responses for change of control voltage of tail motor from 0V to 2V.

$$G_v(s) = \frac{111{,}2}{0{,}3954s^3+0{,}3835s^2+1{,}463s+1} \tag{24}$$

The linear model of the tail motor – azimuth was identified in the same way and the resulting transfer function has the following form:

$$G_h(s) = \frac{700{,}7}{5{,}61s^2+3{,}992s+1} \tag{25}$$

Both models were obtained for the second input set to zero.

Comparison of step responses of the real plant, nonlinear model and liner model is presented in Fig. 5. The courses were obtained for step change of control voltage of main motor from 0V to 1V. Courses obtained for step change of control voltage of the tail motor from 0V to 2V is presented in Fig. 6. In this case the system reached the stopper and bounced back. Bouncing is not modelled and therefore the nonlinear model remained at the stopper.

It can be observed that performance of the enhanced nonlinear model is significantly better than the performances of the other models – the courses of the other models are more distant from the courses of the real-time plant.

Conclusion

The paper presented a model of the Twin Rotor MIMO System – a laboratory model by Feedback. The plant is MIMO system oscillations and nonlinearities present. The model of the plant was designed with respect to its usage in control design of the plant. An initial model based on first principles modelling was derived. This model was enhanced by grey box modelling and designed in Simulink environment. Contrary to original model, the enhanced model contains internal cross-couplings (i.e. influence of the main rotor to the azimuth and influence of the tail rotor to the elevation). Real-time experiments verified a good correspondence of the model and the modelled plant. It can be stated that the main aim of the work – designing a valid model of the Twin Rotor MIMO System, which can be used as a base of the further research – was reached.

Further work will be focused on obtaining even better correspondence of the model with the real plant. Then the model will be used for control design. Several control strategies were presented recently, e.g. predictive control [14] and fuzzy control [15]. Other control strategies – adaptive and predictive control – will be also tested for this plant.

Acknowledgements

The work was performed with financial support of research project NPU I No. MSMT-7778/2014 by the Ministry of Education of the Czech Republic and also by the European Regional Development Fund under the Project CEBIA-Tech No. CZ.1.05/2.1.00/03.0089.

References

[1] V. Bobál, J. Böhm, J. Fessl, J. Macháček:. Digital Self-tuning Controllers: Algorithms, Implementation and Applications, Springer - Verlag London Ltd., London, 2005.
[2] G. P. Liu, Nonlinear identification and control – A neural network Approach, Springer - Verlag London Ltd., London, 2001.
[3] L. Ljung, System identification : theory for the user, Upper Saddle River, NJ : Prentice Hall PTR, 1999.
[4] J. Mikleš, M. Fikar. Process modelling, identification, and control. Springer, Berlin, 2007.
[5] B. Codrons,. Process modelling for control: a unified framework using standard black-box techniques, Springer, London, 2005.
[6] D. M. Himmelblau, J. B. Riggs, Basic principles and calculations in chemical engineering, Upper Saddle River, N.J. : Prentice Hall, 2004.
[7] K.C. Tan, Y. Li, Grey-box model identification via evolutionary computing, Control Engineering Practice 10 (2002) 673–684.
[8] Feedback Instruments, Twin Rotor MIMO System 33-007-PCI, Crowborough, 2013.
[9] Twin Rotor MIMO. Feedback [online]. available from: http://www.feedback-instruments.com/products/education/control_instrumentation/twin_rotor_mimo.
[10] S. Stevanovic, J. Kasac, J.Stepanic, Robust Tracking Control of a Quadrotor Helicopter without Velocity Measurement, Annals of DAAAM for 2012 & Proceedings of the 23rd International DAAAM Symposium (2012), 595 – 600.
[11] Feedback, Twin Rotor Mimo System – Advanced Teaching Manual 1 – 33-007-4M5,
[12] D. Rotondo, F. Nejjari, V. Puig, Quasi-LPV modeling, identification and control of a twin rotor MIMO system, Control Engineering Practice 21 (2013) 829–846.
[13] A. Rahideha, M.H. Shaheed, H. J.C. Huijberts, Dynamic modelling of a TRMS using analytical and empirical approaches, Control Engineering Practice 16 (2008) 241–259.
[14] S. F. Toha, S. Julai, M. O. Tokhi, Ant Colony Based Model Prediction of a Twin Rotor System, Procedia Engineering 41 (2012) 1135 – 1144.
[15] Hanif Ramli, M.S. Meon, T.L.T. Mohamed, A.A.M. Isa, Z. Mohamed, A Fuzzy-Active Force Control Architecture Based in Characterizing Nonlinear Systems' Behavior , Procedia Engineering 41 (2012) 1389 – 1397 .