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Temperature Control in Air-Heating Set via Direct Search Method and Structured Singular Value

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Abstract

The contribution describes an application of the algebraic µ-synthesis to the control of a real plant with a nonlinear characteristic. The controller is designed through algebraic approach and subsequent minimization of the peak of the structured singular value denoted µ. The algebraic approach consists of the pole placement applied to the nominal plant and the algorithm of global optimization Differential Migration treating the problem of multimodality of the cost function. The results are compared with the D-K iteration, synthesis in the ring of proper and stable functions and the Naslin method.

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Keywords: Robust control; structured singular value; algebraic approach; PI and PID control; Differntial Migration; global optimization

1. Introduction

Intensive research activity in the robust control theory during last decades has brought new methods taking into account the parametric, dynamic and mixed structured uncertainties. Some of the methods are based on the $\mathbf{H}_\infty$ approach in the ring of stable and proper transfer functions denoted R_{PS} providing a measure that indicates only the robust stability. Methods based on the Zames' small gain theorem [18] yield both the robust stability and performance conditions. One of them is the structured singular value denoted μ ([8], [16]) treating the robust stability and performance objectives simultaneously. Two methods for the μ-synthesis were derived: the *D-K* iteration [9] and μ-*K* iteration [14]. The *D-K* iteration yields a suboptimal controller minimizing the peak of the upper bound for μ-function. The controller has usually a high order transfer function due to the scaling matrices D, D^{-1} and for further application it is simplified via some kind of approximation. If the simplification is too substantial,

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degradation of the frequency properties of the controller and the whole feedback loop occurs. In some cases, the scaling matrices cannot be approximated with the desired precision and the resulting controller can be far from the optimality. The state-space formulae for the $\mathbf{H}_\infty$ suboptimal controller require the stability of the performance weighting function [10]. The algebraic μ-synthesis ([3], [4], [5], [6], [7]) presented in this contribution overcomes both the approximation of the scaling matrices D, D^{-1} and the impossibility of integrating behaviour of the performance weighting function. The controller is designed through the algebraic pole placement principle applied to the nominal plant and the position of the nominal closed-loop poles is tuned through an evolutionary [2] algorithm with evaluation of the upper bound for μ. The problem of instability of performance weighting function is treated by setting the nominal closed-loop poles to the real axis in the left half-plane.

The control of heating systems has been an important field in the control theory for decades. There are a number of applications of temperature control involving nonlinear and time-delay systems present in the electrical and heating industry as well as in technology processes (e.g. [11] or [15]).

The algebraic μ-synthesis presented in this contribution is more versatile than common methods for the robust control treating the effects of noise, nonlinearity and time delay as well as the influence of external disturbances. The usage of this method is limited to the models fitting in the linear fractional transformation (LFT) framework. In order to fit low order nominal plant to the controlled plant, the parametric uncertainty is used and transformed to LFT interconnection increasing conservatism of the plant model but simplifying the controller.

2. Plant description and identification

The air-heating set considered in this contribution has three input and seven measured quantities. The input signals are the voltage on bulb and the main and adjacent fan. The circuit was controlled by a standard IBM PC computer communicating via serial link (RS232) with the CTRL unit (see Fig. 1). The CTRL unit converts the digital data to unified analogue signals. In the transformation and unification unit, the unified analogue signals are transformed to the voltage on a particular actuator. Similarly, the measured signals are transformed to the unified voltage 0-10 V (Table **Pogreška! Izvor reference nije pronađen.**).

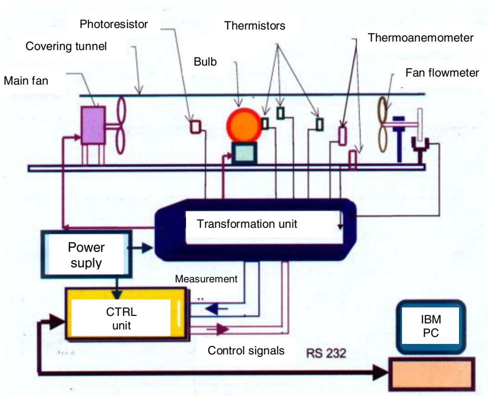

Fig. 1. Plant scheme.

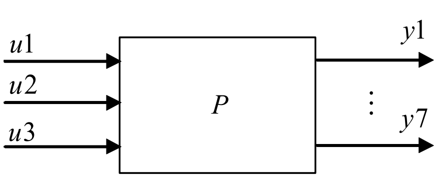

Fig. 2. Inputs and outputs of plant.

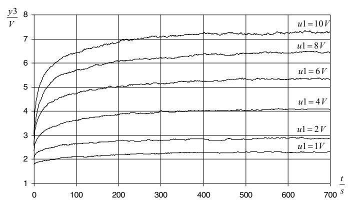

Fig. 3. Step responses of bulb temperature.

Table 1. Input and output channels of CTRL unit.

Inputs	Sensor	Outputs	Actuator
$y1$	sensor of bulb radiance	$u1$	voltage of bulb
$y2$	sensor of temperature near bulb T_2	$u2$	voltage of main fan (speed control)
$y3$	temperature of envelope of the bulb T_3	$u3$	voltage of adjacent fan (speed control)
$y4$	temperature at output of tunnel T_4		
$y6$	thermoanemometer TA_6		
$y7$	fan flowmeter		

The step responses of the measured quantity $y3$ (bulb temperature) for the step of $u1$ (bulb voltage) and constant speed of the main fan ($u2$ = 2V) are depicted in Fig. 3. It is clear from the figure that the step responses have a nonlinear behaviour. The plant is the 2nd order system with relative order 1 which is approximated by the 1st order system. Hence, it should be taken into account that the time constant will vary in a large range. It follows from the steady-state load characteristic that the gain varies in the range of 0.42 to 0.54. The family of transfer function from $u1$ to $y3$ at $u2$ = 2V is:

$$\mathbf{P}_{31} = \left\{ \frac{k}{Ts+1} : k \in (0.42; 0.54), T \in (5; 20) \right\} \tag{1}$$

This means that both the numerator and denominator in (1) are interval polynomials.

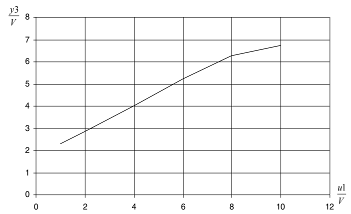

Fig. 4. Steady-state load characteristic.

3. Outline of pole placement design

The pole placement principle is one of the well-known methods for the controller design (e.g. [12], [13], [17]) which is simple for derivation and tuning. Consider a simple feedback loop (1DOF) structure depicted in Fig. 5 with two external inputs – the reference w and disturbance v, respectively. The output and tracking error is according to Fig. 5 in the form

$$y(s) = \frac{bq}{d} w(s) + \frac{cfp}{d} v(s) \tag{2}$$

$$e(s) = w(s) - y(s) = \frac{afp}{d} \cdot \frac{h_w}{f_w} - \frac{cfp}{d} \cdot \frac{h_v}{f_v} \tag{3}$$

where

$$afp + bq = d \tag{4}$$

is the characteristic polynomial of the closed-loop system in Fig. 5.

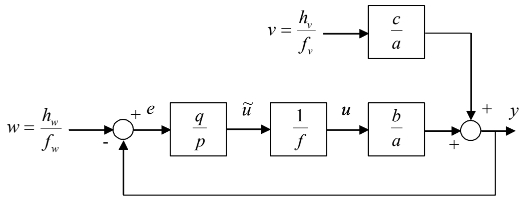

Fig. 5. Structure of 1DOF System.

It can be proven that the asymptotic tracking of the reference is achieved if and only if the polynomial *afp* is divisible by the unstable part of f_w and v is rejected if *cfp* is divisible by the unstable part of f_v so that the result is a finite polynomial. As a consequence, the polynomials p, q are the solutions to Diophantine equation (4). It is also desirable that the transfer function $\frac{q}{fp}$ is proper. Analysis of the polynomial degrees in (4) for the most frequent case $f_w = f = s$ (the stepwise reference) gives

$$\deg d = 2 \deg a \tag{5}$$

A standard choice for the polynomial d is

$$d(s) = \prod_{i=1}^{\deg d} (s + \alpha_i) \tag{6}$$

where $\alpha_i > 0$ are the tuning parameters of the controller and d is a stable polynomial ensuring the internal stability of the nominal system.

With respect to (1), nominal plant transfer function can be expressed by the transfer function

$$P(s) = \frac{b(s)}{a(s)} = \frac{b_0}{s + a_0} \tag{7}$$

and

$$w = \frac{h_w}{f_w} = \frac{1}{s} \tag{8}$$

$$v = \frac{h_v}{f_v} = \frac{1}{s} \tag{9}$$

Then equation (4) has the form

$$(s + a_0)s + b_0(q_1 s + q_0) = s^2 + d_1 s + d_0 \tag{10}$$

and by simple equating the coefficients at the like power of s at the left and right of (10) it can be obtained

$$\begin{aligned} q_1 &= (d_1 - a_0)/b_0 \\ q_0 &= d_0/b_0 \end{aligned} \tag{11}$$

Then the resulting controller is proper and has the traditional PI structure in the form

$$Q = \frac{q_1 s + q_0}{s} \tag{12}$$

4. Principles of μ-Synthesis

The parametric uncertainty in $\widetilde{P}_{31}$ can be treated via the LFT framework using the additive and quotient uncertainty. Define the nominal plant

$$P_{31} = \frac{0.48}{12.5s+1} \tag{13}$$

Plant family $\widetilde{P}_{31}$ is then equivalent to

$$\widetilde{P}_{31}(s,\Delta) = \frac{0.48+0.06\delta_2}{12.5s+1}\frac{1}{1+\dfrac{7.5\delta_3 s}{12.5s+1}}$$

$$\Delta = \begin{bmatrix}\delta_2 & 0\\ 0 & \delta_3\end{bmatrix},\ \delta_2,\delta_3 \in (-1;+1) \tag{14}$$

Let

$$W_2(s) = \frac{0.06}{12.5s+1},\ W_3(s) = \frac{7.5s}{12.5s+1} \tag{15}$$

then

$$\widetilde{P}_{31}(s,\Delta) = (P_{31}+\delta_2 W_2)\frac{1}{1+\delta_3 W_3} \tag{16}$$

Expression (16) is represented by the interconnection depicted in Fig. 6.

The LFT interconnection for μ-synthesis implements performance objectives as well as noise suppression (Fig. 7).

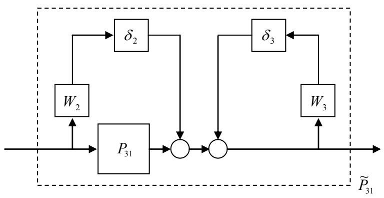

Fig. 6. Structure of 1DOF System.

Performance weight W_1 is defined as

$$W_1(s) = \frac{0.01s+1}{s+0.0001}0.09 \text{ and } \frac{0.01s+1}{s}0.09 \tag{17}$$

for the *D*-*K* iteration and algebraic approach, respectively.

The weight for the *D-K* iteration cannot have integrating behaviour because all weights must be stable causing uncontrollable states in the closed-loop system. The instability and uncontrollability of the closed-loop in Fig. 7 does not make the resulting feedback loop unstable if there is a guarantee that the poles of the nominal feedback loop are in the left half plane. Controllability is a necessary condition for using the state space formulae giving the $\mathbf{H}_\infty$ suboptimal controller [10] as well as the stability of all weighting functions. Thus, it is impossible to use these formulae in this case. The algebraic approach overcomes the problem by setting the nominal closed-loop poles to the left half-plane and by the PI structure

of the controller making possible to use performance weights with the poles at the imaginary axis, which guarantee the asymptotic tracking.

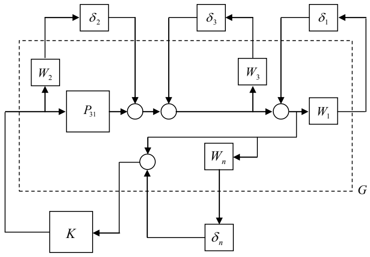

Fig. 7. Structure of 1DOF System.

The weight of noise is a band-pass filter taking into account high frequency noise emerging in sensors

$$W_n(s) = \frac{1/0.001s+1}{1/0.5s+1} 0.001 \tag{18}$$

Let S denote a perturbed transfer function from the reference input w to the tracking error e. Let W_1 denote the weighting function and define the performance condition as

$$\|W_1 S\|_\infty \le 1 \tag{19}$$

If the condition (19) holds then behaviour of the closed loop can be changed through W_1.

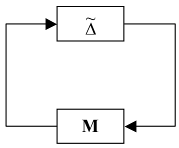

Fig. 8. Transformed Closed-Loop System.

The closed-loop feedback system in Fig. 7 can be transformed to that in Fig. 8, where $\mathbf{M}$ is a linear fractional transformation on $\mathbf{G}(s)$ and controller $K(s)$, i.e.,

$$\mathbf{M} = \mathbf{F}_l(\mathbf{G}, K) = \mathbf{G}_{11} + \mathbf{G}_{12}K(1-G_{22}K)^{-1}\mathbf{G}_{21} \tag{20}$$

where $\mathbf{G}(s)$ is the generalized plant including the nominal plant and weighting functions, which can be parted to

$$\mathbf{G}(s)=\begin{bmatrix}\mathbf{G}_{11} & \mathbf{G}_{12}\\ \mathbf{G}_{21} & G_{22}\end{bmatrix} \tag{21}$$

where the rows and columns of $\mathbf{G}_{11}$ correspond to dynamic perturbations (22) and G_{22} corresponds to the controller structure. The other element of the system is the mixed structured uncertainty forming the diagonal matrix

$$\widetilde{\mathbf{\Delta}}=\{\mathrm{diag}[\delta_1,\delta_2,\delta_3,\delta_n]:\delta_{2,3}\in\mathbf{R},\delta_{1,n}\in\mathbf{C}\} \tag{22}$$

The algebraic μ-synthesis is applied to the 1DOF system for the interconnection depicted in Fig. 7. The *D-K* iteration is applied to the same structure with W_1 without integrating behaviour.

The structured singular value of a matrix **M**, denoted $\mu_{\widetilde{\Delta}}(\mathbf{M})$, is defined as

$$\mu_{\widetilde{\Delta}}(\mathbf{M})=\frac{1}{\min\{\overline{\sigma}(\widetilde{\Delta}):\det(I-\mathbf{M}\widetilde{\Delta})=0\}} \tag{23}$$

and if no such $\widetilde{\Delta}$ exists making $\mathbf{I}-\mathbf{M}\widetilde{\Delta}$ singular, let $\mu_{\widetilde{\Delta}}(\mathbf{M})=0$ [1], where $\overline{\sigma}(\widetilde{\Delta})$ denotes the maximum singular value. The control objective is to find a stabilizing controller **K** minimizing the supremum of $\mu_{\widetilde{\Delta}}(\mathbf{M})$ over all frequencies, i.e.,

$$\min_{\mathbf{K}\,\text{stabilizing}\,\mathbf{G}}\ \sup_{\omega}\mu_{\widetilde{\Delta}}[\mathbf{F}_l(\mathbf{G},\mathbf{K})] \tag{24}$$

The following result is used for the robust performance test [1]:

Theorem 1: The feedback system with $\left|\widetilde{\Delta}\right|<1$, $\widetilde{\Delta}\in\mathbf{C}$ has the robust performance, i.e., expression (19) holds and the perturbed feedback loop is internally stable, if and only if

$$\mu_{\widetilde{\Delta}}(\mathbf{M})\le 1 \tag{25}$$

for all frequencies, where |·| is the absolute value. ■

In the algebraic approach, the nominal closed-loop poles are the tuning parameters and the quality of the controller is evaluated by the upper bound for μ, i.e. as $\inf_{D\in\mathbf{D}}\overline{\sigma}(\mathbf{DMD}^{-1})$ for each frequency. The poles are constraint to the real axis in the left half-plane so that the stability of the nominal feedback loop is guaranteed. As the cost function defined by (24*)* as well as the upper bound has more than one local minimum an algorithm for global optimization is desirable. In this contribution, Differential Migration [2] was used for the optimization with high efficiency in finding the global minimum. Differential Migration is an evolutionary algorithm based on migration of individuals in the space of tuning parameters giving significantly higher robustness (in the sense of ability of finding the global minimum) than other algorithms of this class. The usage of this algorithm can shorten the computational time and increase the probability of finding the optimal pole placement.

5. Control of air-heating set

Experimental studies have been carried out in order to assess the performance of the algebraic μ-synthesis method. The set-point temperature profile provides a reference which comprises 800 iterations. It consists of an initial soak at 3V for 480 iterations followed by a step to 4V which is held constant for 300 iterations. Sampling period is 1 *s* and adjacent fan voltage is kept zero for all experiments.

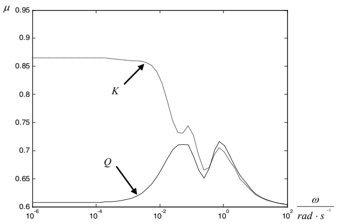

Fig. 9. μ-Plot for *D-K* Iteration (dashed) and Algebraic μ-Synthesis (solid).

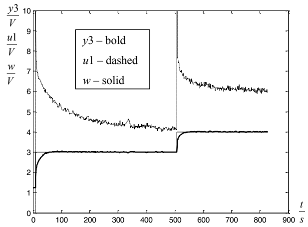

Fig. 10. Control of Real Plant for Algebraic Approach.

The experimental trials are aimed at evaluating the performance of the PI controller obtained via the algebraic μ-synthesis against the *D-K* iteration, synthesis in the R_{PS} with multiple poles and the Naslin method.

Performance indices. In order to draw comparisons between different control schemes, an index or measure of performance is required. The measure of effectiveness which is adopted consists of three factors, these being the amount of energy, the variance of the controlled actuators and the accuracy of set-point tracking. This may be expressed as

$$\in_1 = \frac{\sum u(t)}{\rho} \tag{26}$$

where ρ is the number of iterations.

Increased variance in the control signals to the actuator can lead to correspondingly increased costs consequent upon maintenance and down time due to failure. The variance of the actuator signal may be expressed in the form

$$\in_2 = \sum [u(t) - \in_1]^2 \tag{27}$$

The resulting controller quality arising from control action may be expressed in terms of the accuracy of set-point tracking. Using the integral of absolute error the deviation of the system response $y(t)$ from the set-point $r(t)$ is given as

$$\in_3 = \sum |y(t) - r(t)| \tag{28}$$

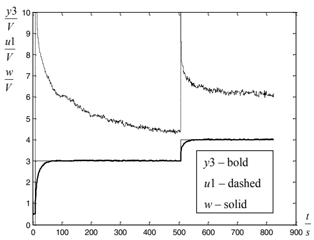

Fig. 11. Control of Real Plant for *D-K* Iteration

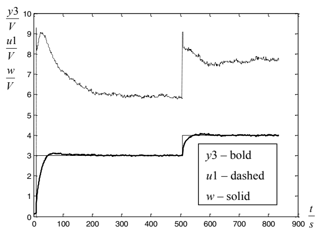

Fig. 12. Control of Real Plant for Synthesis in R_{PS}

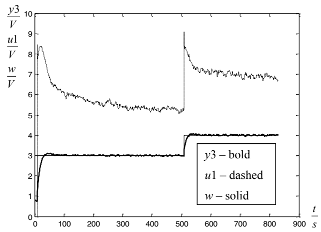

Fig. 13. Control of Real Plant for Naslin Method

In order to provide a basis for comparison, the controllers were tuned to give satisfactory overall performance across the complete temperature profile. The controllers designed in the ring of proper and stable functions

(R_{PS}, see [17]) and by the Naslin method were used as a reference (Fig. 12 and 13). It is clear from Fig. 9, 10, 11, 12, and 13 showing the system response and control input that the algebraic approach produces more favourable results. An improved accuracy in set-point tracking as well as lower energy consumption of the algebraic μ-synthesis (Fig. 10) was achieved via increased control effort to the system resulting in higher variance in the control signal. The set-point, actuator and measured signals are in the unified voltage 0-10V of the CTRL unit. Overall performance indices are given in Table 1.

Table 1. Performance Indices.

Method	$\in_1$	$\in_2$	$\in_3$
Algebraic μ-synthesis	6.30	0.24	8.33
D-K iteration	6.37	0.23	9.07
R_{PS}	7.72	0.17	13.42
Naslin method	7.10	0.22	11.02

Conclusion

The contribution presents an application of the algebraic μ-synthesis to air-heating set, where the temperature of bulb was controlled by its voltage. The controlled system had nonlinear behaviour in the steady-state characteristics. The nonlinearity and the error in approximation of the 2nd order system with the 1st order one was treated via parametric uncertainty and subsequently transformed to the LFT framework. In order to achieve asymptotic tracking, performance weighting function with the pole at the imaginary axis was used. The instability of the nominal feedback loop was treated by setting its poles to the left half-plane via pole placement technique and by choosing the PI structure of the controller treating the unstable pole of the general closed-loop interconnection. The algebraic μ-synthesis was applied to the LFT interconnection including performance weight with integrating behaviour and the results were compared with standard methods for robust controller design - the *D-K* iteration, synthesis in R_{PS} and the Naslin method. Finally, it was shown that the algebraic μ-synthesis had better frequency properties in terms of μ-function and faster set-point tracking than the reference methods.

The algebraic μ-synthesis provides exploitable benefits for a wide range of industrial applications. In contrast to the *D-K* iteration, it can tune simple controllers in a more natural way and guarantee asymptotic tracking, which is desirable in most of the control tasks emerging in technology processes. The only drawback is the fact that an algorithm of global optimization is used leading to longer computational times.

Acknowledgements

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