Adaptive Control of Nonlinear Processes Using Two Methods of Parameter Estimation

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Abstract: – The paper deals with continous-time adaptive control of nonlinear processes. A nonlinear model of the process is approximated by a continuous-time external linear model. The parameters of the CT external linear model of the process are estimated in two ways. In the first case, direct estimation of the CT model is used. In the second case, the parameters of a corresponding delta model are recursively estimated. The control system structure with two feedback controllers is considered. The controller design is based on the polynomial approach. The resulting controllers ensure stability of the control system as well as asymptotic tracking of the step reference and step load disturbance attenuation. The adaptive control is tested on the nonlinear system represented by a model of two spheric liquid tanks in series.

Key-Words: – Adaptive control, continuous-time model, delta model, parameter estimation, polynomial method.

1 Introduction

The most part of technological processes belong to a class of nonlinear systems where both steady-state and dynamic behaviour are nonlinear, e.g. [1], [2]. This fact may cause difficulties when controlling such processes using conventional controllers with fixed parameters.

An effective control of nonlinear processes often requires application of some advanced methods. Here, various efficient methods may be used as various modifications of adaptive control, e.g. [3], predictive control, e.g. [4], [5], robust control, e.g. [6] or nonlinear control, e.g. [7] and [8].

One possible method to cope with this problem is using adaptive strategies based on an appropriate choice of an external linear model (ELM) with recursively estimated parameters. These parameters are consequently used for parallel updating controller‘s parameters.

The control itself can be either continuous-time or discrete. While for design of a continuous-time controller, it is necessary to know a continuous-time ELM and its parameters, a discrete-time controller requires knowledge of a discrete ELM. Experiences of authors in the field of control of nonlinear technological processes indicate that the continuous-time (CT) approach gives better results when controlling processes with strong nonlinearities. In the case of discrete control in order to cope with the nonlinearity, it is necessary to sample signals very frequently. However, it is well known from the properties of transfer functions in the *z*-domain that a sampling period cannot be shortened too much.

Two basic approaches can be used for identification of the continuous-time ELM. The first method is based on filtration of input and output signals where the filtered variables have the same properties (in the *s*-domain) as their non-filtered counterparts, e.g. [9], [10] and [11]. Derivatives of filtered signals that are necessary for the parameters estimate of the CT ELM are obtained from differential filters. This method, presented by authors of this contribution in e.g. [12], has, however, some drawbacks – the necessity to solve additional differential equations representing the filters and to estimate time constants of these filters.

The second strategy uses an external delta model of the controlled process with the same structure as a CT model. The basics of delta models have been described e.g. in [13] and [14]. Although delta models belong into discrete models, they do not have such disadvantageous properties connected with shortening of a sampling period as discrete *z*-models. In addition, parameters of delta models can directly be estimated from sampled signals without the necessity to filter them. Moreover, it can be easily proven that these parameters converge to parameters of CT models for sufficiently small sampling period (compared to the dynamics of the controlled process). A complete description and

experimental verification can be found in [15].
This contribution deals with adaptive control of a non-linear single input – single output process. The parameters of the CT ELM of the process are estimated in two ways. In the first case, direct estimation in terms of filtered variables is used. In the second case, the parameters of a corresponding delta model are recursively estimated.
The control system with two feedback controllers (TFC) is used, see e.g. [16]. This set-up can lead to better control quality than using only one feedback controller. Input signals for the control system are the step reference and step disturbance injected into the input of the controlled process. The resulting controller derived using the polynomial method, e.g. [17] and [18], guarantees stability of the control system, asymptotic tracking of the reference and load disturbance attenuation. The approach is tested on a nonlinear model of two spheric liquid tanks in series.
Many other methods related to solution of similar problems can be found e.g. in [20] – [34].

2 CT External Linear Model

The CT external linear model (ELM) is chosen on the basis of some preliminary knowledge of dynamic behaviour of the controlled nonlinear process. This model is described in the time domain by differential equation

$$a(\sigma)\, y(t) = b(\sigma)\, u(t) \qquad (1)$$

where $\sigma = d/dt$ is the derivative operator and a, b are polynomials in σ. Considering nonzero initial conditions, the ELM is represented in the complex domain by

$$a(s)\, Y(s) = b(s)\, U(s) + o_1(s) \qquad (2)$$

where s is the complex variable and o_1 is the transform of initial conditions. Both a and b now are coprime polynomials in s. The transfer function

$$G(s) = \frac{b(s)}{a(s)} \qquad (3)$$

is considered to be proper (deg b ≤ deg a).

3 Delta Model

Establish the delta operator defined by

$$\delta = \frac{q-1}{T_0} \qquad (4)$$

where q is the forward shift operator and T_0 is the sampling period. When the sampling period is shortened, then, the delta operator approaches the derivative operator σ so that

$$\lim_{T_0 \to 0} \delta = \sigma \qquad (5)$$

and, the δ-model

$$a'(\delta)\, y(t') = b'(\delta)\, u(t') \qquad (6)$$

approaches the continuous-time model (see, e.g. [15]). Here, t' is the discrete time, and, a', b' are polynomials in δ.

4 CT ELM Parameter Estimation

The method of the CT ELM parameter estimation can be briefly carried out as follows.
Since the derivatives of both input and output cannot be directly measured, filtered variables u_f and y_f are established as the outputs of filters

$$c(\sigma)\, u_f(t) = u(t) \qquad (7)$$

$$c(\sigma)\, y_f(t) = y(t) \qquad (8)$$

where $c(\sigma)$ is a stable polynomial in σ that fulfills the condition $\deg c(\sigma) \ge \deg a(\sigma)$. It can be easily proven that the transfer behavior among filtered and among nonfiltered variables are equivalent. Using the L-transform of (7) and (8), the expressions

$$c(s)\, U_f(s) = U(s) + o_2(s) \qquad (9)$$

$$c(s)\, Y_f(s) = Y(s) + o_3(s) \qquad (10)$$

can be obtained where o_2 and o_3 are polynomials of initial conditions. Substituting (9) and (10) into (2), and, after some manipulations, the relation between transforms of the filtered input and output takes the form

$$Y_f(s) = \frac{b(s)}{a(s)} U_f(s) + \Psi(s) \qquad (11)$$

where $\Psi(s)$ is a rational function as the transform of certain function $\psi(t)$ which expresses a difference between initial conditions of filtered and nonfiltered variables (in reference to a last steady state).
Now, the filtered variables including their derivatives can be sampled from filters (7) and (8) in discrete time intervals $t_k = k\,T_S$, k = 0,1,2, ... where T_S is the sampling period. Denoting deg $a = n$ and deg $b = m$, the regression vector can be defined by

$$\boldsymbol{\Phi}^T(t_k) = \Big[-y_f(t_k)\; -y_f^{(1)}(t_k) \ldots\; -y_f^{(n-1)}(t_k) \quad u_f(t_k)\, u_f^{(1)}(t_k) \ldots\; u_f^{(m)}(t_k)\; 1 \Big] \qquad (12)$$

The vector of parameters

$$\boldsymbol{\Theta}^T(t_k) = [a_0\ a_1\ \ldots\ a_{n-1}\ b_0\ b_1 \ldots b_m] \tag{13}$$

and the values of ψ in discrete times can then be estimated from the equation

$$y_f^{(n)}(t_k) = \boldsymbol{\Theta}^T(t_k)\,\boldsymbol{\Phi}(t_k) + \psi(t_k)\,. \tag{14}$$

5 Delta ELM Parameter Estimation

Substituting $t' = k - n$ where $k \geq n$, the equation (6) may be rewriten as

$$\delta^n y(k-n) = b'_m \delta^m u(k-n) + \ldots + b'_1 \delta u(k-n) +$$
$$+ b'_0 u(k-n) - a'_{n-1}\delta^{n-1} y(k-n) - \ldots \tag{15}$$
$$\ldots - a'_{n-1}\delta^{n-1} y(k-n) - \ldots - a'_1 \delta y(k-n) - a'_0 y(k-n)$$

The terms in (15) can be expressed as

$$\delta^i y(k-n) = \sum_{j=0}^{i} \frac{(-1)^j}{T_0^i} \binom{i}{j} y(k-n+i-j) \tag{16}$$

for $i = 0, 1, \ldots, n$, and,

$$\delta^l u(k-n) = \sum_{j=0}^{l} \frac{(-1)^j}{T_0^l} \binom{l}{j} u(k-n+l-j) \tag{17}$$

for $l = 0, 1, \ldots, m$.

Obviously, an actual value of the controlled output $y(k)$ is contained only in the term on the left side of (15) (for $i = n$ in (16)). Now, denoting

$$\varphi_y(k) = \delta^n y(k-n)\,,\ \varphi_y(k-1) = \delta^{n-1} y(k-n)\,, \ldots$$
$$\ldots, \varphi_y(k-n+1) = \delta y(k-n)\,,\ \varphi_y(k-n) = y(k-n)$$
$$\varphi_u(k-n+m) = \delta^m u(k-n)\,, \ldots \tag{18}$$
$$\ldots, \varphi_u(k-n+1) = \delta u(k-n)\,,\ \varphi_u(k-n) = u(k-n)$$

and introducing the regression vector

$$\boldsymbol{\Phi}_\delta^T(k-1) = [-\varphi_y(k-n) - \varphi_y(k-n+1) \ldots \varphi_y(k-1)$$
$$\varphi_u(k-n)\ \varphi_u(k-n+1) \ldots \varphi_u(k-n+m)] \tag{19}$$

then, the parameter vector

$$\boldsymbol{\Theta}_\delta^T = [a'_0\ a'_1 \ldots a'_{n-1}\ b'_0\ b'_1 \ldots b'_m] \tag{20}$$

can be estimated recursively from the regression (ARX) model, see, e.g. [19].

$$\varphi_y(k) = \boldsymbol{\Theta}_\delta^T(k)\,\boldsymbol{\Phi}_\delta(k-1) + \varepsilon(k) \tag{21}$$

where ε(k) is the non-measurable random component.

For a small sampling interval T_0, the estimated parameters reach the parameters of the CT model so that

$$\begin{aligned} b'_j &\to b_j\,, \quad j = 0, 1, \ldots, m \\ a'_i &\to a_i\,, \quad i = 0, 1, \ldots, n-1 \end{aligned} \tag{22}$$

6 Controller Design

The control system with two feedback controllers is depicted in Fig. 1. In the scheme, w is the reference signal, v denotes the load disturbance, e the tracking error, u_0 output of controllers, u the control input and y the controlled output. The transfer function $G(s)$ of the CT ELM is given by (23). The reference w and the disturbance v are considered as

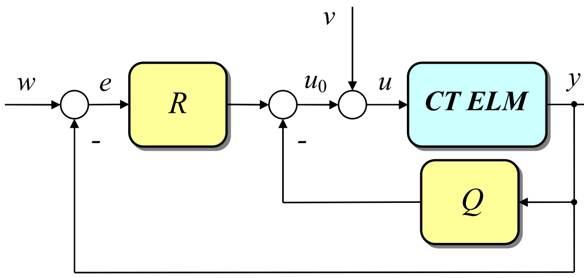

Fig. 1: Control system with two feedback controllers

the step functions with transforms

$$W(s) = \frac{w_0}{s}\,,\quad V(s) = \frac{v_0}{s}\,. \tag{23}$$

The transfer functions of both controllers are in forms

$$R(s) = \frac{r(s)}{\tilde{p}(s)}\,,\quad Q(s) = \frac{\tilde{q}(s)}{\tilde{p}(s)} \tag{24}$$

where $\tilde{q}$, r and $\tilde{p}$ are coprime polynomials in s fulfilling the condition of properness $\deg r \leq \deg \tilde{p}$ and $\deg q \leq \deg \tilde{p}$.

The controller design described in this section appears from the polynomial approach. The general requirements on the control system are formulated as its internal properness and stability, asymptotic tracking of the reference and load disturbance attenuation. The procedure to derive admissible controllers can briefly be performed as follows:

Let the polynomial t has the form

$$t(s) = r(s) + \tilde{q}(s)\,. \tag{25}$$

Then, the control system stability is ensured when polynomials $\tilde{p}$ and t are given by a solution of the polynomial equation

$$a(s)\tilde{p}(s)+b(s)t(s)=d(s) \qquad (26)$$

with a stable polynomial d on the right side. Evidently, the roots of d determine the closed-loop poles.
Taking into account the transform of the tracking error

$$E(s)=\frac{1}{d}\left[(a\tilde{p}+b\tilde{q})W(s)-b\tilde{p}V(s)\right] \qquad (27)$$

and both transforms (23), the asymptotic tracking and load disturbance attenuation are provided by polynomials $\tilde{p}$ and $\tilde{q}$ having the form

$$\tilde{p}(s)=s\,p(s)\,,\quad \tilde{q}(s)=s\,q(s)\,. \qquad (28)$$

Subsequently, the transfer functions (24) take forms

$$Q(s)=\frac{q(s)}{p(s)}\,,\quad R(s)=\frac{r(s)}{s\,p(s)} \qquad (29)$$

and, a stable polynomial $p(s)$ in their denominators ensures the stability of controllers.
Now, the polynomial t can be rewritten to the form

$$t(s)=r(s)+s\,q(s)\,. \qquad (30)$$

Taking into account the solvability of (26) and the condition of internal properness, the degrees of polynomials in (26) and (29) can be easily derived as

$$\begin{array}{c}\deg t=\deg r=\deg a,\quad \deg q=\deg a-1\\ \deg p\geq\deg a-1,\quad \deg d\geq 2\deg a\end{array}\,. \qquad (31)$$

Denoting deg a = n, polynomials t, r and q have forms

$$t(s)=\sum_{i=0}^{n}t_i s^i\,,\ r(s)=\sum_{i=0}^{n}r_i s^i\,,\ q(s)=\sum_{i=1}^{n}q_i s^{i-1} \qquad (32)$$

and, relations among their coefficients are

$$r_0=t_0\,,\ \ r_i+q_i=t_i\ \text{ for } i=1,\ldots,n\,. \qquad (33)$$

Since by a solution of the polynomial equation (26) provides calculation of coefficients t_i, unknown coefficients r_i and q_i can be obtained by a choice of selectable coefficients $\beta_i\in\langle 0,1\rangle$ such that

$$r_i=\beta_i t_i\,,\ \ q_i=(1-\beta_i)t_i\ \text{ for } i=1,\ldots,n\,. \qquad (34)$$

The coefficients β_i distribute a weight between numerators of transfer functions Q and R.
Remark: If $\beta_i=1$ for all i, the control system in Fig. 1 reduces to the 1DOF control configuration (Q = 0). If $\beta_i=0$ for all i, and, both reference and load disturbance are step functions, the control system corresponds to the 2DOF control configuration.

7 Example

Two spheric liquid tanks in series are considered according to Fig. 2.

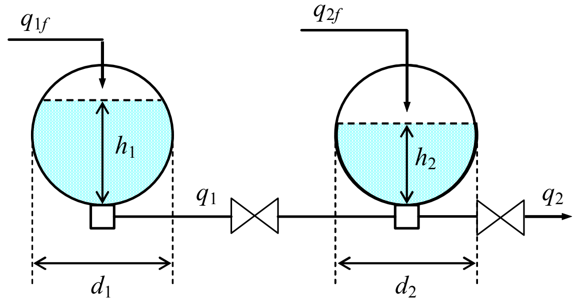

Fig. 2: Two spheric liquid tanks in series.

Using standard simplifications, the model of the plant can be described by two nonlinear differential equations

$$\pi h_1(d_1-h_1)\frac{dh_1}{dt}+q_1=q_{1i} \qquad (35)$$

$$\pi h_2(d_2-h_2)\frac{dh_2}{dt}-q_1+q_2=q_{2i} \qquad (36)$$

where d_j are diameters of tanks, h_j are liquid levels in tanks, q_j are stream flowrates and q_{jf} are their inlet values, (for j = 1, 2). The stream volumetric flowrates depend upon levels in tanks as

$$q_1=k_1\sqrt{|h_1-h_2|} \qquad (37)$$

$$(\text{ if } h_1-h_2<0 \text{ then } q_1=-q_1\,)$$

$$q_2=k_2\sqrt{h_2} \qquad (38)$$

where k_1, k_2 are constants.
Initial conditions for (42), (43) are steady state liquid levels $h_1(0)=h_1^s$, $h_2(0)=h_2^s$. The model parameters and values of variables at the operating point used in simulations are in Tab. 1.

Table 1: Parameters and steady-state values.

$k_1=1.5\,\mathrm{m}^{2.5}\,\mathrm{min}^{-1}$	$k_2=0.8\,\mathrm{m}^{2.5}\,\mathrm{min}^{-1}$
$h_1^s=1.631\,\mathrm{m}$	$h_2^s=1.381\,\mathrm{m}$
$d_1=d_2=3\mathrm{m}$	
$q_{1f}^s=0.75\,\mathrm{m}^3\,\mathrm{min}^{-1}$	$q_{2f}^s=0.19\,\mathrm{m}^3\,\mathrm{min}^{-1}$

The task is to control of the liquid levels in the first or in the second tank. In both cases, the control input is the inlet flow to the first tank. For the control purposes, the controlled outputs and control input variables are considered to be deviations from their values at the operating point as

$$\begin{aligned} y_1(t) &= h_1(t) - h_1^s \\ y_2(t) &= h_2(t) - h_2^s \end{aligned} \qquad (39)$$

$$u(t) = q_{1f}(t) - q_{1f}^s \qquad (40)$$

Steady-state characteristics of liquid levels in both tanks are in Figs. 3 and 4.

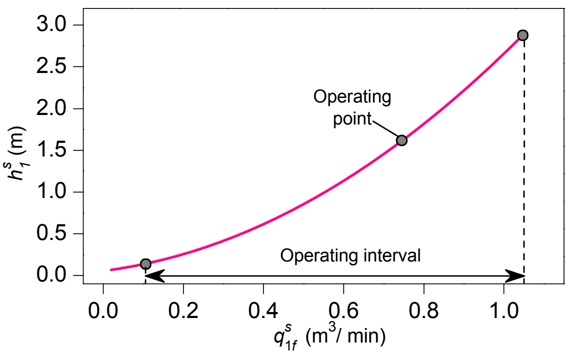

Fig. 3: Dependence of liquid level in the first tank on the input flow.

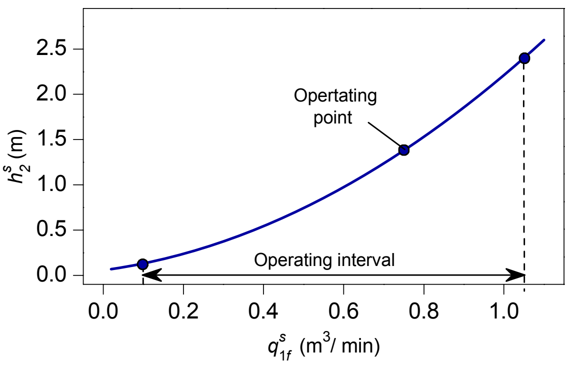

Fig. 4: Dependence of liquid level in the second tank on the input flow.

The values of the input and both outputs defining the operating interval are in Table 2.

Table 2: Boundary of the operating interval

$q_{1f}^{\min} = 0.1$ m^3/min	$q_{1f}^{\max} = 1.05$ m^3/min
$h_1^{\min} = 0.136$ m	$h_1^{\max} = 2.893$ m
$h_2^{\min} = 0.131$ m	$h_2^{\max} = 2.403$ m

The second order CT ELMs have been chosen for both tanks on the basis of simulated controlled output step responses shown in Figs. 5 and 6.

The CT ELM for the first tank takes in the time domain the form of the differential equation

$$\ddot{y}_1(t) + a_1 \dot{y}_1(t) + a_0 y_1(t) = b_1 \dot{u}(t) + b_0 u(t) \qquad (41)$$

and, in the complex domain the form of the transfer function

$$G(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} \qquad (42)$$

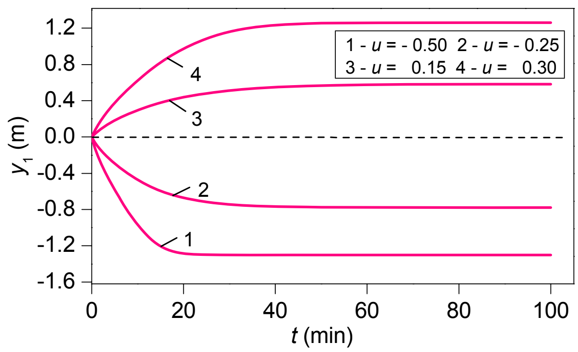

Fig. 5: The first controlled output step response.

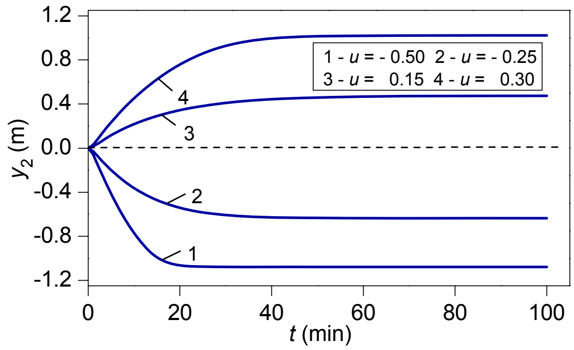

Fig. 6: The second controlled output step response.

For the second tank, the corresponding descriptions have forms

$$\ddot{y}_2(t) + a_1 \dot{y}_2(t) + a_0 y_2(t) = b_0 u(t) \qquad (43)$$

and,

$$G(s) = \frac{b_0}{s^2 + a_1 s + a_0} \qquad (44)$$

In next control procedures, the delta ELM parameter estimation is used for the first tank, and, the direct CT ELM parameter estimation for the second tank.

The delta ELM accordant with (41) has the form

$$\begin{aligned}&\delta^2 y_1(t') + a_1' \delta y_1(t') + a_0' y_1(t') = \\ &= b_1' \dot{u}(t') + b_0' u(t')\end{aligned} . \qquad (45)$$

Now, the regression vector (20) takes the form

$$\begin{aligned}\boldsymbol{\Phi}_\delta^T(k-1) = \big(&\varphi_u(k-1)\ \varphi_u(k-2) \\ &-\varphi_y(k-1) - \varphi_y(k-2)\big)\end{aligned} \qquad (46)$$

with elements

$$\begin{aligned}\varphi_y(k-2) &= y(k-2) \\ \varphi_y(k-1) &= \frac{y(k-1) - y(k-2)}{T_0} \\ \varphi_u(k-2) &= u(k-2) \\ \varphi_u(k-1) &= \frac{u(k-1) - u(k-2)}{T_0}\end{aligned} \qquad (47)$$

and, the vector of the delta model parameters

$$\boldsymbol{\Theta}_\delta^T(k) = \left[b_1' \ b_0' \ a_1' \ a_0' \right] \qquad (48)$$

can be recursively estimated from the ARX model

$$\varphi_y(k) = \boldsymbol{\Theta}_\delta^T(k) \boldsymbol{\Phi}_\delta(k-1) + \varepsilon(k) \qquad (49)$$

where

$$\varphi_y(k) = \frac{y(k) - 2y(k-1) + y(k-2)}{T_0^2} . \qquad (50)$$

The recursive estimation of delta model parameters was performed with the sampling interval T_0 = 0.2 min.

In the direct CT ELM parameter estimation, the filtered variables and their derivatives are obtained from filters

$$\ddot{y}_{2f}(t) + c_1 \dot{y}_{2f}(t) + c_0 y_{2f}(t) = y_2(t) \qquad (51)$$

$$\ddot{u}_f(t) + c_1 \dot{u}_f(t) + c_0 u_f(t) = u(t) \qquad (52)$$

with filter parameters c_1 = 1, c_0 = 0.25.

Then, the CT ELM parameters $[b_0, a_1, a_0]$ are recursively estimated from the ARX model

$$\begin{aligned}\ddot{y}_{2f}(t_k) = b_0 u_f(t_k) - a_1 \dot{y}_{2f}(t_k) - \\ - a_0 y_{2f}(t_k) + \psi(t_k)\end{aligned} \qquad (53)$$

in discrete time intervals $t_k = k\ T_s$ with the sampling period T_s = 1 min.

In both cases, the recursive identification method with exponential and directional forgetting by course of [19] was used.

For the second order model (44) with $\deg a = 2$, the controller's transfer functions take specific forms

$$\begin{aligned}Q(s) &= \frac{q(s)}{p(s)} = \frac{q_2 s + q_1}{s + p_0} \\ R(s) &= \frac{r(s)}{s\,p(s)} = \frac{r_2 s^2 + r_1 s + r_0}{s(s + p_0)}\end{aligned} . \qquad (54)$$

where

$$\begin{aligned}&r_0 = t_0, \quad r_1 = \beta_1 t_1, \quad r_2 = \beta_2 t_2 \\ &q_1 = (1-\beta_1) t_1, \quad q_2 = (1-\beta_2) t_2\end{aligned} \qquad (55)$$

The controller parameters then result from a solution of the polynomial equation (26) and depend upon coefficients of the polynomial d. The next problem here is to find a stable polynomial d that enables to obtain acceptable stabilizing controllers.

In this paper, the polynomial d with roots determining the closed-loop poles is chosen as

$$d(s) = n(s)(s + \alpha)^2 \qquad (56)$$

where n is a stable polynomial obtained by spectral factorization

$$a^*(s) a(s) = n^*(s) n(s) \qquad (57)$$

and α is the selectable parameter.

Note that a choice of d in the form (56) provides the control of a good quality for aperiodic controlled processes.

The coefficients n then are expressed as

$$n_0 = \sqrt{a_0^2}\ , \quad n_1 = \sqrt{a_1^2 + 2n_0 - 2a_0} \qquad (58)$$

and, the controller parameters p_0 and t can be obtained from solution of the matrix equation

$$\begin{pmatrix} 1 & b_1 & 0 & 0 \\ a_1 & b_0 & b_1 & 0 \\ a_0 & 0 & b_0 & b_1 \\ 0 & 0 & 0 & b_0 \end{pmatrix} \times \begin{pmatrix} p_0 \\ t_2 \\ t_1 \\ t_0 \end{pmatrix} = \begin{pmatrix} d_3 - a_1 \\ d_2 - a_0 \\ d_1 \\ d_0 \end{pmatrix} \qquad (59)$$

for the first tank, and

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ a_1 & b_0 & 0 & 0 \\ a_0 & 0 & b_0 & 0 \\ 0 & 0 & 0 & b_0 \end{pmatrix} \times \begin{pmatrix} p_0 \\ t_2 \\ t_1 \\ t_0 \end{pmatrix} = \begin{pmatrix} d_3 - a_1 \\ d_2 - a_0 \\ d_1 \\ d_0 \end{pmatrix} \qquad (60)$$

for the second tank, where

$$\begin{aligned}&d_3 = n_1 + 2\alpha, d_2 = 2\alpha n_1 + n_0 + \alpha^2 \\ &d_1 = 2\alpha n_0 + \alpha^2 n_1, d_0 = \alpha^2 n_0\end{aligned} \qquad (61)$$

Now, it follows from the above introduced procedure that tuning of controllers can be performed by a suitable choice of selectable parameters β and α.
The controller parameters r and q can then be obtained from (45).
Control systems using delta ELM and CT ELM parameter estimation are depicted in Figs. 7 and 8.

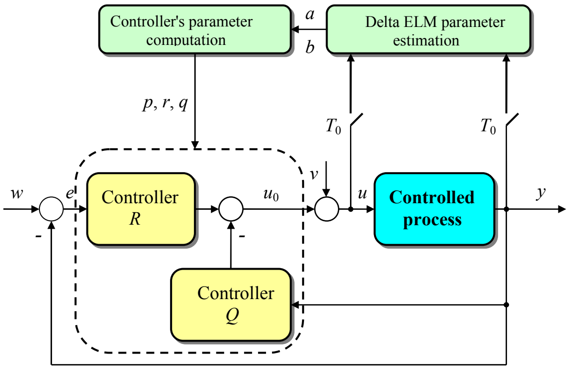

Fig 7: Adaptive control system with delta ELM parameter estimation.

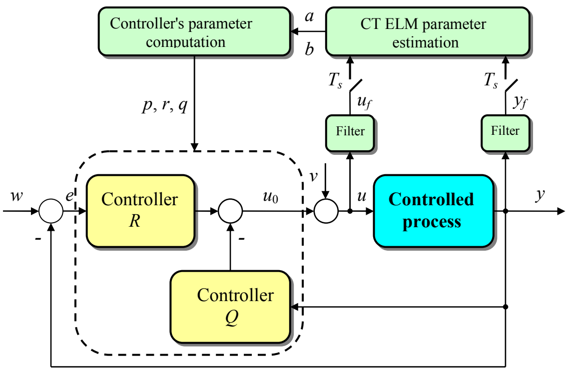

Fig 8: Adaptive control system with CT ELM parameter estimation.

8 Simulations

For the control of the liquid level in the first tank, the delta ELM parameter estimation was used, and, in the second tank, the direct CT ELM parameter estimation was applied.
For the start (the adaptanion phase) in all simulations, the P controller with a small gain was used.
The controlled output responses in the first tank obtained for two values of α are shown in Fig. 9.. There is a minimal difference between both responses, but both responses exhibit a small overshoot. A greater difference is reflected in control input responses shown in Fig. 10. There, a small change in a selection of α leads to significant changes of the control input.

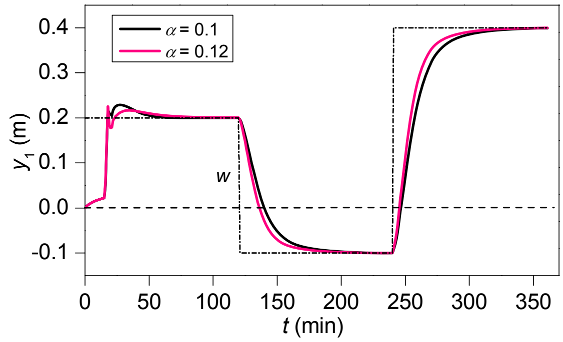

Fig. 9: Controlled output responses (delta ELM) ($\beta_2 = \beta_1 = 1$).

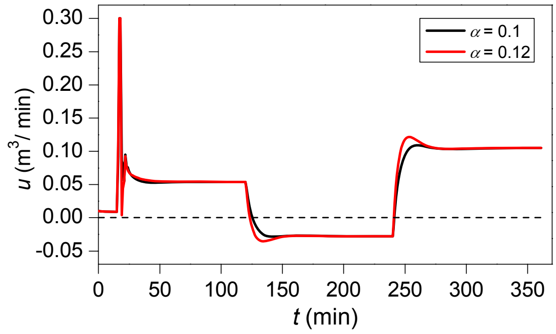

Fig. 10: Control iput responses (delta ELM PE) ($\beta_2 = \beta_1 = 1$).

The controlled output and control input responses in the second tank computed for three values of α are shown in Figs. 11 and 12.. In this case, the responses without overshoots can be obtained by a choice of a satisfactory α.

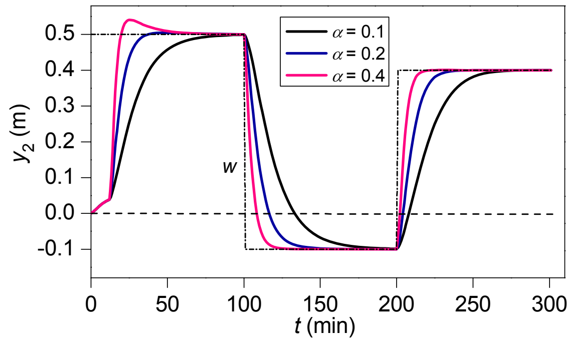

Fig. 11: Controlled output responses (CT ELM) ($\beta_2 = \beta_1 = 1$).

Controlled output responses in the second tank in

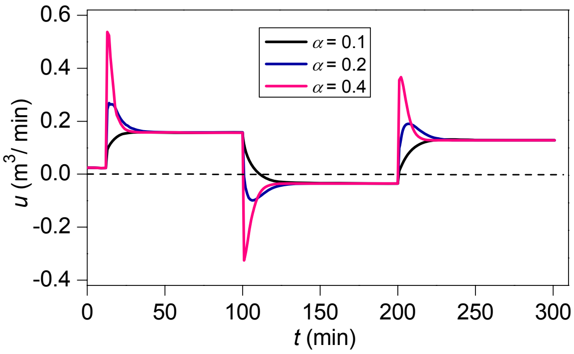

Fig. 12: Control input responses (CT ELM) ($\beta_2 = \beta_1 = 1$).

Fig. 13 show the possibility to obtain courses without overshoots also by a selection of suitable parameters β. An effect of these parameters on the control input is evident from Fig. 14.

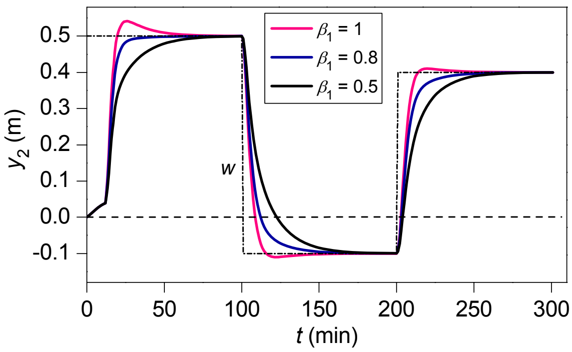

Fig. 13: Controlled output responses (CT ELM) ($\beta_2 = 0$, $\alpha = 0.4$).

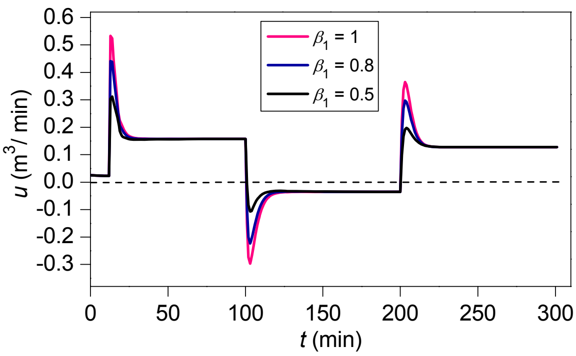

Fig. 14: Control input responses (CT ELM) ($\beta_2 = 0$, $\alpha = 0.4$).

The evolution of the delta and CT ELM parameters during control can be seen in Figs. 15 and 16. It can be seen that in both cases parameters quickly stabilize after each change of the reference signal.
A presence of the integrating part in controller transfer functions enables step disturbance attenuation. The controlled output responses to a setpoint reference and step disturbances for both tanks is shown in Figs. 17 and 18. Here, filtered step changes $v = \Delta q_{2f}$ were injected into the system. The controller parameters were estimated only in the first (tracking) interval. Then, the controller with fixed parameters was used.

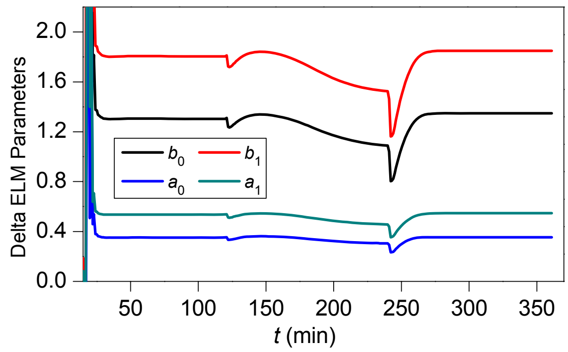

Fig. 15: Delta ELM parameter evolution ($\beta_2 = \beta_1 = 1$, $\alpha = 0.1$)

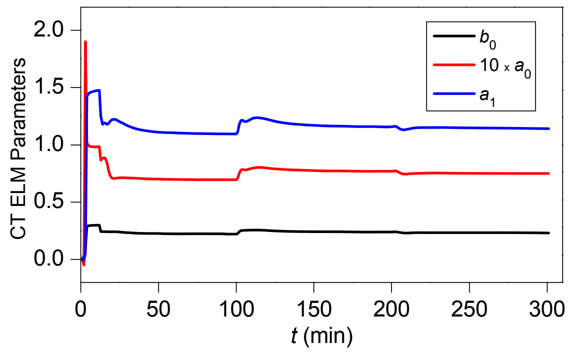

Fig. 16: CT ELM parameter evolution ($\beta_2 = \beta_1 = 1$, $\alpha = 0.2$)

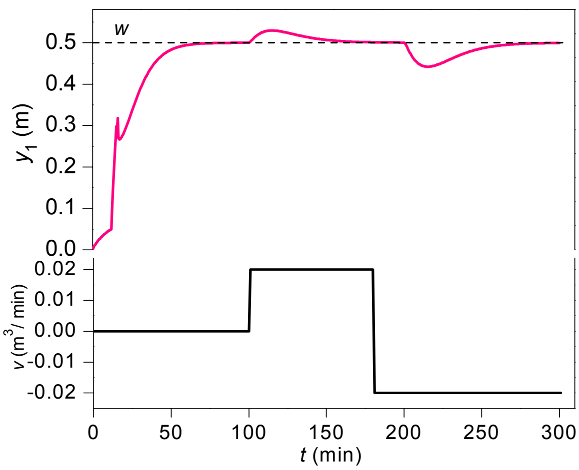

Fig. 17: The first tank – disturbance attenuation.

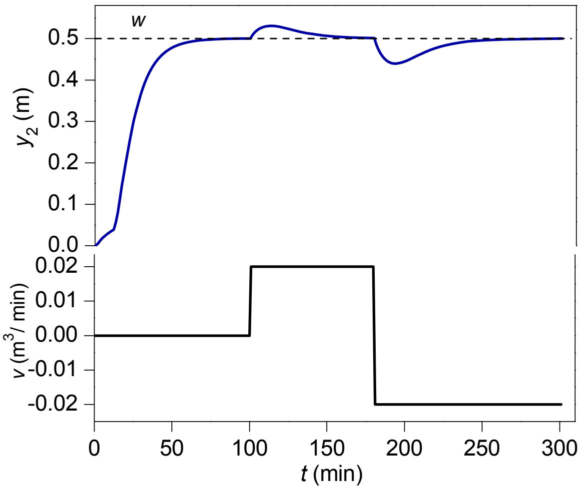

Fig. 16: The second tank – disturbance attenuation.

9 Conclusions

In this paper, one possible approach to the continuous-time adaptive control of a nonlinear process was proposed. The presented strategy enables to create an effective control algorithm. This algorithm is based on an alternative continuous-time external linear model with parameters recursively estimated in two ways. In the first case, direct estimation of the CT model in terms of filtered variables is used. In the second case, the parameters of a corresponding delta model are recursively estimated. Both parts of the resulting continuous-time controller are given by a solution of polynomial Diophantine equations. Parameters of the controller are periodically readjusted according to recursively estimated parameters of the CT or delta model. The controller parameters may be tuned by a single selectable parameter. The presented method has been tested by computer simulation on the nonlinear model of two spheric tanks in series. The results demonstrate the applicability of the presented control strategy. It can be deduced that the described adaptive strategy is also suitable for other technological processes.

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References:

[1] B.A. Ogunnake, and W.H. Ray, *Process dynamics, modelling, and control*, Oxford University Press, New York, 1994.

[2] R. Smith, Chemical process design and integration, John Wiley & Sons, Chichester, 2005.

[3] P. Ioannou, and B. Fidan, Adaptive control tutorial, SIAM, Philadelphia, 2006.

[4] M. Kvasnica, M. Herzeg, Ľ. Čirka, and M. Fikar, Model predictive control of a CSTR: A hybrid modelling approach, *Chemical Papers*, Vol. 64, 2010, pp. 301–309.

[5] L. Magni, G. De Nicolao, L. Magnani, and R. Scattolini, A stabilizing model-based predictive control algorithm for nonlinear systems, *Automatica*, Vol. 37, 2001, pp. 1351–1362.

[6] M. Bakošová, D. Puna, P. Dostál, and J. Závacká, Robust stabilization of a chemical reactor, *Chemical Papers*, Vol. 63, 2009, pp. 527–536.

[7] D. Astolfi, D. Karagiannis, and R. Ortega, *Nonlinear and adaptive control with applications*, Springer Verlag, London, 2008.

[8] P. Dostál, M. Bakošová, J. Vojtěšek, and V. Bobál, Adaptive nonlinear control of a continuous stirred tank reactor, *Chemical Papers*, Vol. 65, 2011, pp. 636-643.

[9] L. Garnier, L. Wang, and P.C. Young, Direct identification of continuous-time models from sampled data: Issues, basic solutions and relevance, in *Identification of continuous-time models from sampled data* (*eds. H. Garnier and L. Wang*), Springer Verlag, London, 2008.

[10] G.P. Rao, and H. Unbehauen, Identification of continuous-time systems, *IEE Proc.-Control Theory Appl.*, Vol. 153, 2006, pp. 185-220.

[11] B. Wahlberg, The effects of rapid sampling in system identification, *Automatica*, Vol. 26, 1990, pp. 167-170.

[12] P. Dostál, V. Bobál, and M. Blaha, One approach to adaptive control of nonlinear processes, in *Proc. of the IFAC Workshop on Adaptation and Learning in Control and Signal Processing ALCOSP 2001*, pp. 407-412, Cernobbio-Como, Italy, 2001.

[13] R.H. Middleton, and G.C. Goodwin, *Digital Control and Estimation - A Unified Approach*, Prentice Hall, 1990.

[14] S. Mukhopadhyay, A.G. Patra, and G.P. Rao, New class of discrete-time models for continuos-time systems, *International Journal of Control*, Vol. 55, 1992, pp. 1161-1187.

[15] D.L. Stericker, and N.K. Sinha, Identification of continuous-time systems from samples of input-output data using the δ-operator, *Control-Theory and Advanced Technology*, Vol. 9, 1993, pp. 113-125.

[16] Dostál, F. Gazdoš, V. Bobál, and J. Vojtěšek, Adaptive control of a continuous stirred tank reactor by two feedback controllers, in *Proc. 9th IFAC Workshop Adaptation and Learning in Control and Signal Processing ALCOSP'2007*, Saint Petersburg, Russia, 2007, pp. P5-1 – P5-6.

[17] V. Kučera, Diophantine equations in control – A survey, *Automatica*, Vol. 29, 1993, pp. 1361-1375.

[18] M.J. Grimble, and V. Kučera (eds.), *Polynomial methods for control systems design*, Springer-Verlag, London, 1996.

[19] V. Bobál, J. Böhm, J. Fessl, and J. Macháček, *Digital self-tuning controllers*, Springer-Verlag, Berlin, 2005.

[20] A.V. Doroshin, and F. Neri, Open research issues on nonlinear dynamics: Dynamical systems and processes, *WSEAS Transactions on Systems*, Vol. 13, 2014, in press,.

[21] C. Ciufudean, and F. Neri, Open research issues on Multi-Models for complex technological systems, *WSEAS Transactions on Systems*, Vol. 13, 2014, in press.

[22] F. Neri, Open research issues on computational techniques for financial applications, *WSEAS Transactions on Systems*, Vol. 13, 2014, in press.

[23] P. Karthikeyan, and F. Neri, Open research issues on deregulated electricity market: investigation and solution methodologies, *WSEAS Transactions on Systems*, Vol. 13, 2014, in press.

[24] M. Panoiu, and F. Neri, Open research issues on modeling, simulation and optimization in electrical systems, *WSEAS Transactions on Systems*, Vol. 13, 2014, in press.

[25] F. Neri, Open research issues on advanced control methods: Theory and application, *WSEAS Transactions on Systems*, Vol. 13, 2014, in press.

[26] P. Hájek, and F. Neri, An introduction to the special issue on computational techniques for trading systems, time series forecasting, stock market modeling, financial assets modeling, *WSEAS Transactions on Business and Economics*, Vol. 10 (4), 2013, pp. 201-292.

[27] M. Azzouzi, and F. Neri, An introduction to the special issue on advanced control of energy systems, *WSEAS Transactions on Power Systems*, Vol. 8 (3), 2013, pp. 103.

[28] Z. Bojkovic, and F. Neri, An introduction to the special issue on advances on interactive multimedia systems, *WSEAS Transactions on Systems*, Vol. 12 (7), 2013, pp. 337-338.

[29] L. Pekař, and F. Neri, An introduction to the special issue on advanced control methods: Theory and application, *WSEAS Transactions on Systems*, Vol. 12 (6), 2013, pp. 301-303.

[30] C. Guarnaccia, and F. Neri, An introduction to the special issue on recent methods on physical polluting agents and environment modeling and simulation, *WSEAS Transactions on Systems*, Vol. 12 (2), 2013, pp. 53-54.

[31] F. Neri, An introduction to the special issue on computational techniques for trading systems, time series forecasting, stock market modeling, and financial assets modeling, *WSEAS Transactions on Systems*, Vol. 11 (12), 2012, pp. 659-660.

[32] M. Muntean, and F. Neri, Foreword to the special issue on collaborative systems, *WSEAS Transactions on Systems*, Vol. 11 (11), 2012, pp. 617.

[33] L. Pekař, and F. Neri, An introduction to the special issue on time delay systems: Modelling, identification, stability, control and applications, *WSEAS Transactions on Systems*, Vol. 11 (10), 2012, pp. 539-540.

[34] C. Volos, and F. Neri, An introduction to the special issue: Recent advances in defense systems: Applications, methodology, technology, *WSEAS Transactions on Systems*, Vol. 11 (9), 2012, pp. 477-478.