Self-tuning Control of Non-linear Servomotor: Standard Versus Dual Approach

VLADIMÍR BOBÁL[1,2], PETR CHALUPA[2], PETR DOSTÁL[1,2], MAREK KUBALCÍK[2]
[1]Centre of Polymer Systems, University Institute
[2]Department of Process Control, Faculty of Applied Informatics, Tomas Bata University in Zlín
T. G. Masaryka 5555, 760 01 Zlín
CZECH REPUBLIC
bobal@fai.utb.cz, http://web.fai.utb.cz/cs/docs/CV_english.pdf

Abstract: - The majority of processes met in the industrial practice have stochastic characteristics and eventually they embody non-linear behaviour. Traditional controllers with fixed parameters are often unsuitable for such processes because their parameters change. The changes of process parameters are caused by changes in the manufacturing process, in the nature of the input materials, fuel, machinery use (wear) etc. Fixed controllers cannot deal with this. One possible alternative for improving the quality of control for such processes is the use of adaptive control systems. Different approaches were proposed and utilized. One successful approach is represented by self-tuning controller (STC). This approach is also called system with indirect adaptation (with direct identification). The main idea of an STC is based on the combination of a recursive identification procedure and a selected controller synthesis. Presently, most of the STCs are based on the Certainty Equivalence (CE) Principle, which is only suboptimal. One of the possibilities to improve the quality of these adaptive control methods is usage of an Adaptive Dual Control (the bicriterial approach). In this paper, the bicriterial approach is verified and compared with some other adaptive control approaches based on the CE Principle by the real-time control of a highly non-linear laboratory model, the DR300 Speed Control with Variable Load.

Key-Words: - Self-tuning control; Dual control; Bicriterial approach; ARX model; Recursive least squares; Non-linear system; Servo motor; Real-time control

1 Introduction

One approach to adaptive control is based on recursively estimating the unknown system characteristics, gradually specifying them, and then monitoring possible changes. Using this knowledge, appropriate methods can be employed to design the optimal controller. This kind of controllers, which identifies unknown processes and then synthesizes control law (adaptive control with recursive identification), is referred to in the literature as a *self-tuning controller* (STC) – see e. g. [1] – [5]. This approach is generally suitable also for a control of the time delay systems [6] – [9]. The self-tuning [10], [11], auto-tuning [12], predictive [13] and adaptive predictive [14] control of the time-delay systems is designed in the above mentioned papers.

It is clear that to reach these goals the identification of the static and dynamic characteristics of a controlled process plays an important role, together with the optimal control strategy itself. It is known from parameter estimation theory that the determination of parameters is always burdened by a degree of uncertainty or error. This uncertainty not only depends on the number of identification steps (i.e., the amount of sampled data) and the choice of structure for the mathematical model of the controlled process, but is also dependent on the behaviour of the controller output, the sampling period, and the choice of filter for the controller and process outputs. This means that every realized change in the controller output except the required control result also excites the controlled system and thus creates the condition for its identification; in other words, for the best identification of the controlled process, it is necessary to impose certain conditions on the sequence of controller inputs.

The controller output signal of an optimal adaptive system should have two main properties:

- It must ensure that the process output follows the reference signal value and responds to its changes.
- It must excite the controlled process sufficiently to allow identification.

These properties are introduced in the literature as *dual properties* (or *dual features*) and adaptive control systems with these two properties are called *adaptive dual control systems*.

The exact solution to the optimal dual adaptive control problem was presented by Feldbaum [15], [16] using dynamic programming. Unfortunately, because of the complexity of calculations it involves, exact dual optimal control is too demanding to be of use in most situations.

It has, therefore, been necessary to simplify the solution of this problem. This solution is based on the constrained separation of identification and control, the *Certainty Equivalence* (CE) *Principle*. The basic principle of CE is that the model uncertainty is not considered. For the controller design the parameter estimates of the process model, which are obtained by recursive identification, are used. It is assumed at the same time that values of these estimates correspond to their actual values. It is obvious that adaptive control systems based on the CE approach are not always optimal. For that purpose, several simplified approaches to the design of adaptive dual control systems have been developed. These simplifications can be divided into two main groups based on: (1) *approximations* of the dual problem, known as *implicit dual control methods*; and (2) *reformulation* of the problem, known as *explicit dual control methods* [17] - [20].

In [21], a simulation study of four different suboptimal dual controllers for time-varying stochastic systems is presented. A two-stage dual suboptimal controller for stochastic systems and a multistage suboptimal dual controller using optimal predictors are presented in [22] and [23]. An active SubOptimal Dual controller (ASOD) [24] has been examined in simulation conditions. Murray-Smith and Sbarbaro [25] designed a non-linear adaptive controller using non-parametric Gaussian process prior models. This controller has dual features, both tracking a reference signal and learning a model of the system from the observed responses. A neural adaptive dual controller with dynamic structure for non-linear stochastic systems was derived by Šimandl and Král [26]. The model of the stochastic system is based on a Gaussian radial basis function neural network; the dual controller design is based on two separate criteria. Paper [27] is concerned with a dual approach to start-up of an adaptive predictive controller in case of *a priori* unknown plants. In [28] the dual adaptive control for linear system with unknown constant parameters is given.

One of the most efficient approaches is given by the *bicriterial synthesis* method for dual adaptive controllers. The main idea of the bicriterial approach is the introduction of two cost functions that correspond to the two goals of dual control: (1) to track the plant output to the desired reference signal; and (2) to introduce excitations to improve the parameter estimation. This bicriterial approach was developed for discrete systems essentially by Filatov and Unbehauen [29]. The dual version of the direct adaptive pole-placement controller using bicriterial optimization was designed in [30]. Indirect adaptive dual control for Hammerstein systems using artificial neural networks is presented in [31]. This controller can be applied to non-linear systems by simulation and provides excellent prospects for industry. There are also other successful methods of modelling nonlinear systems such as an agent based modelling [32].

In this paper, the bicriterial approach is used for adaptive dual control of the DR300 servo laboratory model (speed control with variable load) in real-time conditions and is compared with three standard STCs that are based on the CE principle. These three controllers differ only in the algorithm of recursive identification (different approach to forgetting procedure). The usage of the adaptive algorithms for control of an above-cited laboratory model is motivated by a substantial uncertainty of its dynamic behaviour and a range of non-linearities that result from its static characteristics (see Section 4).

This paper is organized in the following way. Section 2 gives a brief view of the structure of adaptive dual control and the bicriterial approach. Dual modification of CE STC is the content of Section 3. The description of DR300 laboratory servo model and analysis of its static and dynamic properties are introduced in Section 4. Section 5 is devoted to the real-time control of the non-linear servo system together with an evaluation of experimental verifications. Section 6 concludes the paper.

2 Structure of Adaptive Dual Control and Bicriterial Approach

The main difference between a conventional CE adaptive control system (see Fig. 1) and an adaptive dual control system (see Fig. 2) lies in the parameter estimates transmission. In the dual system, both parameter estimates and their accuracy are considered. If the uncertainty of the recursively acquired parameter estimates is taken into account, it is possible to calculate the controller output, which ensures the optimal excitation of the system for quality identification while keeping the cautious character of the controlling signal. This approach can markedly improve the quality of control of systems with limited *a priori* information and high levels of uncertainty.

The two-criteria minimization method, called the bicriterial approach, is based on sequential minimization of two cost functions for dual control corresponding to the two aims of dual control (see

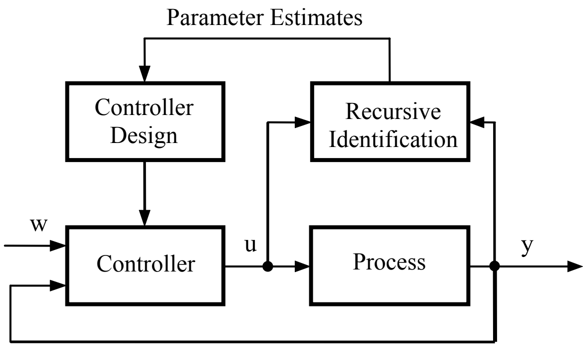

Fig. 1. CE adaptive control system

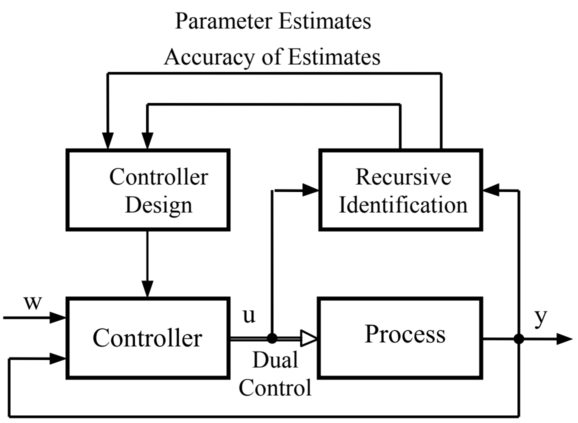

Fig. 2. Dual adaptive control system

Fig. 3). The first function is control losses J_k^c and its optimum after minimization is the *cautious control* action $u_c(k)$. This cautious controller results in a control signal with a smaller magnitude than that of an ordinary CE controller, so there are smaller overshoots after the start of a process. The second cost function, J_k^a, which represents the parametric uncertainty, is minimized around the cautious control value in the Ω_k domain. The resulting control action value is a compromise of optimization of the two criteria when the magnitude of the excitation is given by the size of the domain Ω_k. It is appropriate to define these constraints symmetrically around the cautious control value $u_c(k)$ by the value of parameter θ_k representing the magnitude of the additional excitations. Finally, we obtain the dual controller by bicriterial optimization

$$u(k) = \underset{u(k)\in\Omega_k}{\operatorname{argmin}} J_k^{\mathrm{a}} \tag{1}$$

$$\Omega_k = \left[u_{\mathrm{c}}(k) - \theta(k);\ u_{\mathrm{c}}(k) + \theta(k)\right] \tag{2}$$

$$\theta(k) = \eta \operatorname{tr}\{\boldsymbol{C}(k)\};\quad \eta \geq 0 \tag{3}$$

$$u_{\mathrm{c}}(k) = \underset{u(k)}{\operatorname{argmin}} J_k^{\mathrm{c}} \tag{4}$$

The amplitude of excitations is dependent on the value of the selectable parameter η and the trace of the covariance matrix $\boldsymbol{C}(k)$.

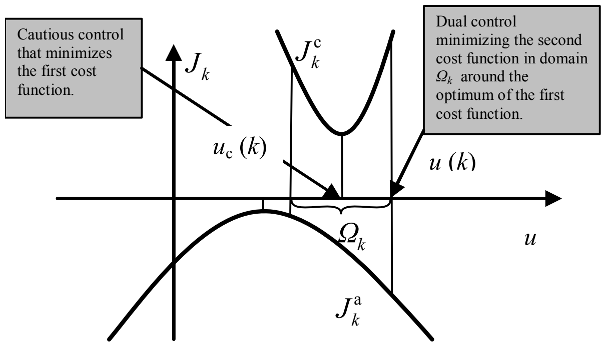

Fig. 3. Optimization of two cost functions

3 Dual Modification of CE Controller

With an explicit STC, it is possible to use the design of a dual controller independently of the structure of the standard CE adaptive controller. A dual controller obtained in this way can be used together with any CE controller with indirect adaptation (e.g., pole assignment, LQG, digital Ziegler–Nichols, predictive, or generalized minimum variance). It is introduced as an additional unit modifying the CE control signal to the dual control one. This simple modification improves the control performance.

Now consider a single input–single output system described by the linear stochastic differential equation (discrete time input/output model)

$$\begin{aligned} y(k+1) &= b_1 u(k) + \ldots + b_{nb} u(k - n_b + 1) - a_1 y(k) \\ &\quad - \ldots - a_{na} y(k - n_a + 1) + n(k) \\ &= b_1 u(k) + \boldsymbol{\Theta}_0^T \boldsymbol{\Phi}_0(k) + n(k) \\ &= \boldsymbol{\Theta}^T \boldsymbol{\Phi}(k) + n(k) \end{aligned} \tag{5}$$

Provided that $n_a = n_b = n$ is

$$\boldsymbol{\Theta}^T = \left[b_1,...,b_n,a_1,...,a_n\right] = \left[b_1 \vdots \boldsymbol{\Theta}_0^T\right] \tag{6}$$

the ARX model parameter vector and

$$\begin{aligned}\boldsymbol{\Phi}^T(k) &= \left[u(k),...,u(k-n+1),-y(k),...,-y(k-n+1)\right] \\ &= \left[u(k) \vdots \boldsymbol{\Phi}_0^T(k)\right]\end{aligned} \tag{7}$$

is the regression vector ($y(k)$ is the process output variable, and $u(k)$ is the controller output variable). The noise sequence $n(k)$ has variance σ_n^2. A simple *recursive least squares* identification method is used to estimate the plant parameters. The vector of parameter estimates is updated as

$$\hat{\boldsymbol{\Theta}}(k+1) = \hat{\boldsymbol{\Theta}}(k) + \frac{\boldsymbol{C}(k)\boldsymbol{\Phi}(k)}{\boldsymbol{\Phi}^T(k)\boldsymbol{C}(k)\boldsymbol{\Phi}(k)+\sigma_n^2}\hat{e}(k+1) \tag{8}$$

where

$$\hat{e}(k+1) = y(k+1) - \hat{\boldsymbol{\Theta}}^T(k)\boldsymbol{\Phi}(k) \tag{9}$$

is the prediction error. The square covariance matrix is updated in each sampling period according to

$$\boldsymbol{C}(k+1) = \boldsymbol{C}(k) - \frac{\boldsymbol{C}(k)\boldsymbol{\Phi}(k)\boldsymbol{\Phi}^T(k)\boldsymbol{C}(k)}{\boldsymbol{\Phi}^T(k)\boldsymbol{C}(k)\boldsymbol{\Phi}(k)+\sigma_n^2} \tag{10}$$

The following notation for the covariance matrix is introduced for later manipulations

$$\begin{aligned}\boldsymbol{C}(k) &= E\left\{\left[\boldsymbol{\Theta}-\hat{\boldsymbol{\Theta}}(k)\right]\left[\boldsymbol{\Theta}-\hat{\boldsymbol{\Theta}}(k)\right]^T \middle| \Im_k\right\} \\ &= \begin{bmatrix} c_{11}(k) & \dots & c_{1n}(k) \\ \vdots & \ddots & \vdots \\ c_{n1}(k) & \dots & c_{nn}(k)\end{bmatrix} = \begin{bmatrix} c_{b_1}(k) & \boldsymbol{c}_{b_1\boldsymbol{\Theta}_0}^T(k) \\ \boldsymbol{c}_{b_1\boldsymbol{\Theta}_0}(k) & \boldsymbol{C}_{\boldsymbol{\Theta}_0}(k)\end{bmatrix}\end{aligned} \tag{11}$$

The set of process outputs and inputs available at time k is denoted as

$$\begin{aligned}\Im_k &= \left\{y(k),...,y(0),u(k-1),...,u(0)\right\} \\ k &= 1,...,N-1; \quad \Im_0 = \left\{y(0)\right\}\end{aligned}$$

The nominal system output for the CE controller is

$$\hat{y}(k+1) = \hat{b}_1(k)u_{\mathrm{CE}}(k) + \hat{\boldsymbol{\Theta}}_0^T(k)\boldsymbol{\Phi}_0(k) \tag{12}$$

where $u_{\mathrm{CE}}(k)$ is the CE controller output signal. Dual control cost functions are given as

$$J_k^{\mathrm{c}} = E\left\{\left[\hat{y}(k+1)-y(k+1)\right]^2 \middle| \Im_k\right\} \tag{13}$$

and

$$J_k^{\mathrm{a}} = -E\left\{\left[y(k+1)-\hat{\boldsymbol{\Theta}}^T(k)\boldsymbol{\Phi}(k)\right]^2 \middle| \Im_k\right\}. \tag{14}$$

Substituting equations (5) and (12) into equation (13) and minimizing the modified equation (13) leads to the cautious control law

$$u_{\mathrm{c}}(k) = \frac{\hat{b}_1^2(k)u_{\mathrm{CE}}(k) - \boldsymbol{c}_{b_1\boldsymbol{\Theta}_0}^T(k)\boldsymbol{\Phi}_0(k)}{\hat{b}_1^2(k)+c_{b_1}(k)}. \tag{15}$$

Criterion (1) with the constraints given by equation (2) can be directly minimized leading to

$$\begin{aligned}u(k) &= u_c(k) \\ &+ \theta(k)\operatorname{sgn}\left\{J_k^{\mathrm{a}}\left[u_{\mathrm{c}}(k)-\theta(k)\right] - J_k^{\mathrm{a}}\left[u_{\mathrm{c}}(k)+\theta(k)\right]\right\}\end{aligned} \tag{16}$$

and after a final modification, the resulting dual control law is in the form [6]

$$u(k) = u_{\mathrm{c}}(k) + \theta(k)\operatorname{sgn}\left\{c_{b_1}(k)u_{\mathrm{c}}(k) + c_{b_1\boldsymbol{\Theta}_0}^T(k)\boldsymbol{\Phi}_0(k)\right\}$$

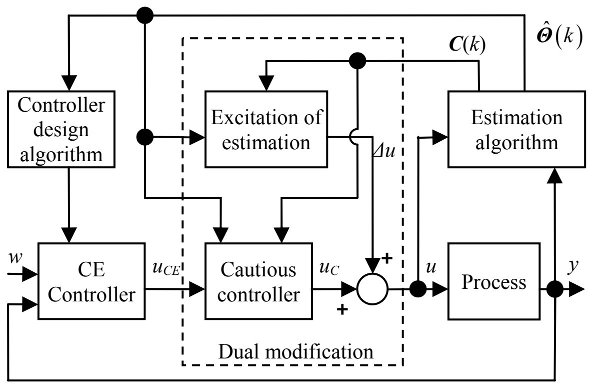

Fig. 4. Detailed scheme of an adaptive dual control system (Δu represents the optimal excitation)

(17)

The detailed structure of an adaptive control system with a dual control unit is shown in Fig. 4.

4 Real-time Control of a Non-linear System

The proposed dual control algorithm was tested using a real-time laboratory model DR300 (Speed Control with Variable Load) by the Amira Company, Duisburg, Germany (see Fig. 5). A block scheme of the DR300 system is presented in Fig. 6.

The plant is represented by a permanently exited DC - motor (M1) of which the input signal (armature current) is provided by a current control loop. Its servo amplifier operates in 4 quadrant mode, so that the orientation of the current and correspondingly the orientation of the rotation of the motor is arbitrarily adjustable. The sensors for the output signal (speed) are a tachogenerator (T) and an incremental encoder (I). The free end of the motor shaft is fixedly coupled (K) to the shaft of a generator. Its output current is freely adjustable. The rotation speed of the motor M1 is driven by voltage u. The motor shaft rotations per minute (rpm) are measured by tachogenerator T. The aim of the control process is to control the rotation speed of the shaft ω by the control voltage u. From the control point of view, the Amira DR300 is a non-linear system. Some characteristics of the nonlinearity (gain with dead zone and hysteresis) can be observed from the static characteristics shown in Fig. 7.

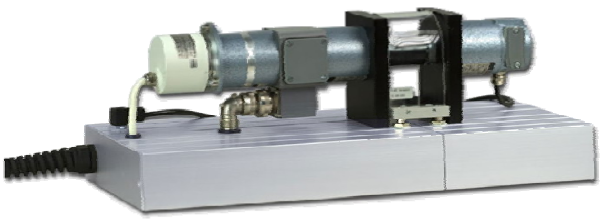

Fig. 5. Laboratory model Amira DR300

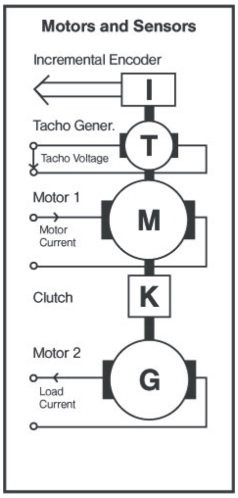

Fig. 6. Block scheme of Amira DR300 servomotor.

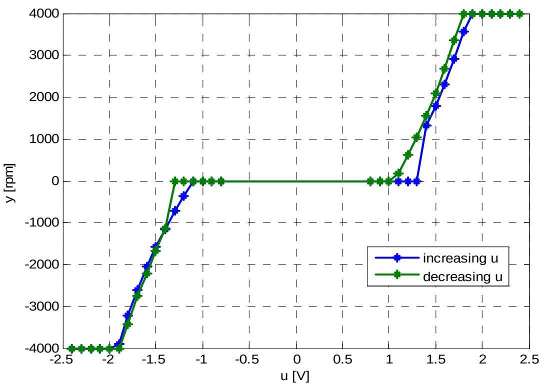

Fig. 7. Static characteristics of Amira DR300 servomotor

Even in the parts of static characteristics, where the plant output changes (approximately -2V to -1V and 1V to 2V), the gain of the system is not constant. The gain of the plant varies from approximately 3600 rpm/V to 6900 rpm/V.

A servo motor control is being solved e.g. in [33] – [36]. The disturbance rejection of a nonlinear servo system using the self-tuning control is described in [37].

Four different adaptive control algorithms were used to control the DR300 system. All the controllers were based on the pole assignment approach in two-degrees-of-freedom (2DOF) configurations (see Fig. 8).

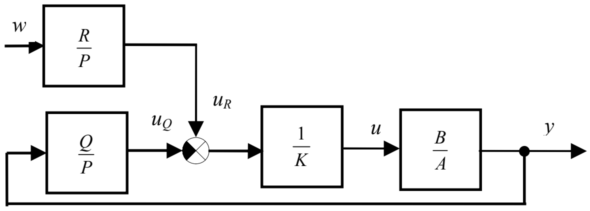

Fig. 8. Closed-loop 2DOF control system

The controlled system was modelled using a discrete second-order linear ARX model

$$A\left(z^{-1}\right)y(k)=B\left(z^{-1}\right)u(k)+n(k) \qquad (18)$$

where

$$A\left(z^{-1}\right)=1+a_1z^{-1}+a_2z^{-2}$$
$$B\left(z^{-1}\right)=b_1z^{-1}+b_2z^{-2} \qquad (19)$$

and the non-measurable random component $n(k)$ is assumed to have zero mean value $E[n(k)] = 0$ and constant covariance (dispersion) $\sigma_n = E[n^2(k)]$.

Polynomials of the feedback control part

$$G_R(z)=\frac{Q\left(z^{-1}\right)}{P\left(z^{-1}\right)} \qquad (20)$$

were computed by solving the characteristic closed-loop polynomial

$$A(z^{-1})K(z^{-1})P(z^{-1})+B(z^{-1})Q(z^{-1})=D\left(z^{-1}\right) \qquad (21)$$

where

$$K\left(z^{-1}\right)=1-z^{-1}. \tag{22}$$

The feedback controller polynomials and the desired polynomial have the form

$$P\left(z^{-1}\right)=1+p_1z^{-1};\quad Q\left(z^{-1}\right)=q_0+q_1z^{-1}+q_2z^{-2} \tag{23}$$

$$D\left(z^{-1}\right)=1+d_1z^{-1}+d_2z^{-2}. \tag{24}$$

By solving equation (21), a system of linear equations can be obtained using the method of indeterminate coefficients

$$\begin{bmatrix} \hat{b}_1 & 0 & 0 & 1 \\ \hat{b}_2 & \hat{b}_1 & 0 & \hat{a}_1-1 \\ 0 & \hat{b}_2 & \hat{b}_1 & \hat{a}_2-\hat{a}_1 \\ 0 & 0 & \hat{b}_2 & -\hat{a}_2 \end{bmatrix}\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ p_1 \end{bmatrix}=\begin{bmatrix} d_1+1-\hat{a}_1 \\ d_2+\hat{a}_1-\hat{a}_2 \\ \hat{a}_2 \\ 0 \end{bmatrix} \tag{25}$$

where $\hat{a}_1$, $\hat{a}_2$, $\hat{b}_1$, and $\hat{b}_2$ are current estimates of the process model parameters.

From the matrix equation (25) it is possible to compute the feedback controller parameters. The coefficients of polynomial (24) were chosen as $d_1=-1.6, d_2=0.64$.

Polynomial $R\left(z^{-1}\right)$ of the feed forward control part

$$G_R(z)=\frac{R\left(z^{-1}\right)}{P\left(z^{-1}\right)} \tag{26}$$

was computed by solving the polynomial equation

$$D_w(z^{-1})S(z^{-1})+B(z^{-1})R(z^{-1})=D(z^{-1}) \tag{27}$$

where $S\left(z^{-1}\right)$ is the auxiliary polynomial. For a step change of the reference value $w(k)$, $D_w\left(z^{-1}\right)=1-z^{-1}$, and it is then possible to solve equation (27) by substituting $z=1$.

$$r_0=\frac{D(1)}{B(1)}=\frac{1+d_1+d_2}{\hat{b}_1+\hat{b}_2} \tag{28}$$

From Fig. 8 it is obvious that the controller is given by the equation

$$P(z^{-1})K(z^{-1})u(k)=R(z^{-1})w(k)-Q(z^{-1})y(k) \tag{29}$$

and the CE control law is then

$$\begin{aligned} u_{CE}(k)&=r_0w(k)-q_0y(k)-q_1y(k-1) \\ &-q_2y(k-2)+(1-p_1)u(k-1)+p_1u(k-2) \end{aligned} \tag{30}$$

A sampling period of T_0 = 0.05 s was used in all experiments, and initial parameter estimates were set without using *a priori* information about the controlled system:
$\hat{a}_1(0)=0.1$, $\hat{a}_2(0)=0.2$, $\hat{b}_1(0)=0.3$, $\hat{b}_2(0)=0.4$ and the initial value of the covariance matrix $C(0)=10^9 I$ was used.

The first controller used was the dual controller described in the previous sections. This controller is further referenced as *dual*. The individual vectors and parameters in equations (15) and (17) have the form

$$\boldsymbol{\Phi}_0^T(k)=\left[u(k),y(k),-y(k-1)\right];$$

$$\boldsymbol{c}_{b_1\boldsymbol{\Theta}_0}^T=\left[c_{12}(k),c_{13}(k),c_{14}(k)\right];\ c_{b_1}(k)=c_{11}(k)$$

(see the covariance matrix (11)); the selectable parameter chosen for equation (3) was $\eta=30$.

The other three controllers were taken from the Self-tuning Controllers Simulink Library [5], [38], [39]. The controller structure *pp2chp* was used. These controllers only differed in the algorithm for recursive identification.

The first one used the pure Recursive Least Squares Method (RLSM) [40] and is referenced as *lsm*.

The second one contained RLSM with an exponential forgetting factor [41] and is referenced as *ef*.

The third one used RLSM, extended to include the technique directional (adaptive) forgetting [5], [42], [43] and is referenced as *adf*. In this case the vector of parameter estimates is updated according to the recursive relation

$$\hat{\boldsymbol{\Theta}}(k)=\hat{\boldsymbol{\Theta}}(k-1)+\frac{\boldsymbol{C}(k-1)\boldsymbol{\Phi}(k)}{1+\xi(k)}\hat{e}(k) \tag{31}$$

where

$$\hat{\boldsymbol{\Theta}}^T(k)=\left[\hat{a}_1,\hat{a}_2,\hat{b}_1,\hat{b}_2\right]$$

$$\boldsymbol{\Phi}^T(k)=\left[-y(k-1),-y(k-2),u(k-1),u(k-2)\right] \tag{32}$$

are the vector of parameter estimates and the data (regression) vector.

The expression

$$\xi(k)=\boldsymbol{\Phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{\Phi}(k) \tag{33}$$

is an auxiliary scalar, and

$$\hat{e}(k)=y(k)-\hat{\boldsymbol{\Theta}}^T(k-1)\boldsymbol{\Phi}(k) \tag{34}$$

is the prediction error. If $\xi(k) > 0$, the square covariance matrix is updated by the relation

$$\boldsymbol{C}(k)=\boldsymbol{C}(k-1)-\frac{\boldsymbol{C}(k-1)\boldsymbol{\Phi}(k)\boldsymbol{\Phi}^T(k)\boldsymbol{C}(k-1)}{\varepsilon^{-1}(k)+\xi(k)} \quad (35)$$

where

$$\varepsilon(k)=\varphi(k)-\frac{1-\varphi(k)}{\xi(k-1)} \quad (36)$$

If $\xi(k-1)=0$, then

$$\boldsymbol{C}(k)=\boldsymbol{C}(k-1). \quad (37)$$

The value of adaptive directional forgetting $\varphi(k)$ is then calculated for each sampling period as

$$[\varphi(k)]^{-1}=1+(1+\rho)\left[\ln\left(1+\xi(k)\right)\right] + \left[\frac{\left(\upsilon(k)+1\right)\eta(k)}{1+\xi(k)+\eta(k)}-1\right]\frac{\xi(k)}{1+\xi(k)} \quad (38)$$

where

$$\eta(k)=\frac{\hat{e}^2(k)}{\lambda(k)}; \qquad \upsilon(k)=\varphi(k-1)\left[(\upsilon(k-1)+1\right]$$

$$\lambda(k)=\varphi(k-1)\left[\lambda(k-1)+\frac{\hat{e}^2(k)}{1+\xi(k)}\right] \quad (39)$$

are auxiliary variables.

The closed loop stability is one of the problems that are not satisfactory solved for the STC. Usage of a suitable recursive identification method and related achievement of unbiased and convergent parameter estimates are very important to ensure the closed loop stability. The problem of unbiasedness and convergence of parameter estimates of STC has been considered as an important and very difficult mathematical problem. This problem was not investigated for a long time because of the difficulties connected with nonlinearity and complexity of the adaptive control laws. If the unbiasedness and convergence of parameter estimates is guaranteed, STCs operate without problem. It is also to be taken into consideration that for processes described with adequate lower order model the large tolerance exists for the model inaccuracy. In case of the control of the laboratory model DR300, the second order model is used which contributes to better stability of the closed loop. Because the pole assignment method is used in the control part of the control loop, stability can be provided by an appropriate selection of the coefficients of the desired polynomial $D\left(z^{-1}\right)$ (24).

General problem of the convergence and stability of adaptive dual controllers is investigated and stability conditions are derived in [44].

4.1 Controllers' Performance

The control performance of the DR300 system controlled by controller *dual* is shown in Fig. 9. It can be seen that the control process quickly became stable and the parameters of the model reached values sufficient for the asymptotic tracking of the reference signal.

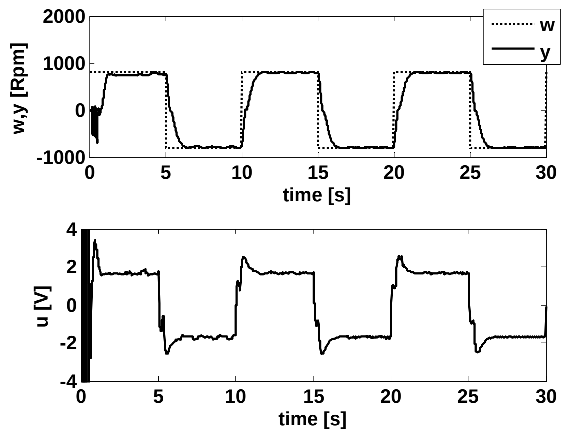

Fig. 9. Control of DR300 using *dual* controller

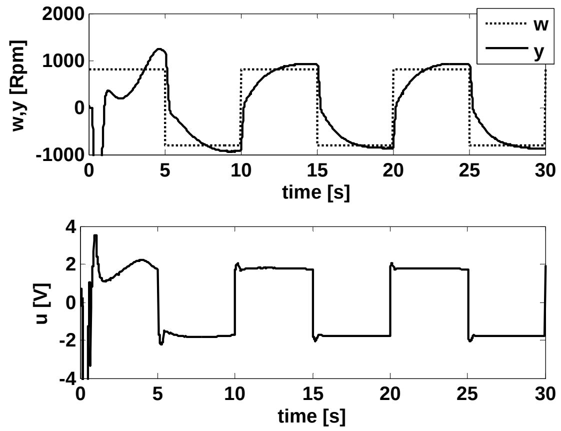

Fig. 10. Control of DR300 using *lsm* controller

The *lsm* controller used pure RLSM for identification of the controlled system. All input–output pairs affect the parameters of the model by the same weight. As can be seen in Fig. 10, the *lsm* controller is not able to cope with the DR300 control problem as accurately as the *dual* controller.

The identification based on the RLSM *with exponential forgetting* is used in the *ef* controller.

The forgetting coefficient of $\varphi = 0.95$ was used for the control of the DR300 system. The performance of the control loop signals is shown in Fig. 11. It can be seen that after about 3 s the parameters produced by recursive identification became good enough to make the control loop stable with asymptotic tracking of the reference signal. Oscillations of both controller and process signal output occur when the reference signal changes. This behaviour is caused by the different model parameters for the positive and negative values of process signal output (see the static characteristic in Fig. 7).

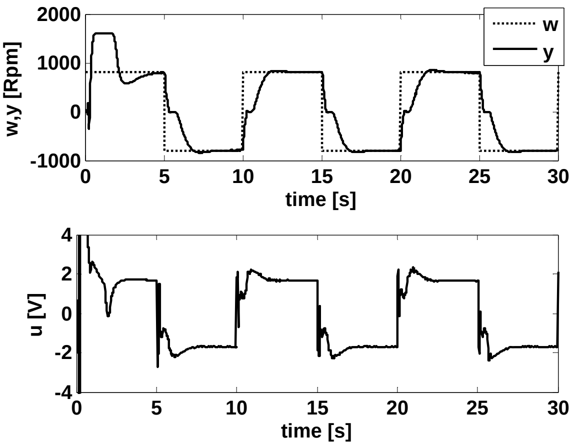

Fig. 11. Control of DR300 using *ef* controller

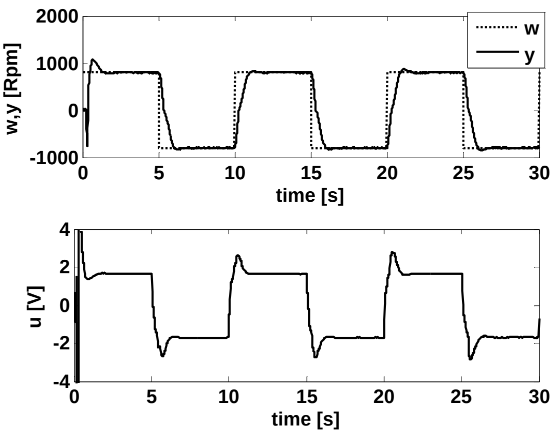

Fig. 12. Control of DR300 using *adf* controller

4.2 Comparison of Control Performance Using Summing Criteria

The performances of the individual controllers were compared not only by investigating graphs of controller performance and process output signals, but also mathematical criteria. Four criteria were used to compare the control results obtained by individual controllers

$$S_{e2} = \frac{1}{b-a+1}\sum_{k=a}^{b}\left[w(k)-y(k)\right]^2 \tag{40}$$

$$S_{ea} = \frac{1}{b-a+1}\sum_{k=a}^{b}\left|w(k)-y(k)\right| \tag{41}$$

$$S_{u2} = \frac{1}{b-a}\sum_{k=a}^{b-1}\left[u(k+1)-u(k)\right]^2 \tag{42}$$

$$S_{ua} = \frac{1}{b-a}\sum_{k=a}^{b-1}\left|u(k+1)-u(k)\right| \tag{43}$$

Values of individual criteria are shown in Table 1.

Criteria S_{e2} and S_{ea} are based on control error. The sum of squares of control error and the sum of absolute values of control error were used to obtain S_{e2} and S_{ea} respectively. These criteria represent the accuracy of the control process. Criteria S_{u2} and S_{ua} are based on changes to the control signal. The sum of squares of the control sequence and the sum of absolute values of the control sequence were used to obtain S_{u2} and S_{ua} respectively. These criteria represent demands for actuators. Values of a and b were selected to cover the whole control process except the first 3 s.

Table 1. Values of criteria for the control results

controller	$S_{e2}\cdot 10^{-3}$	S_{ea}	$S_{u2}\cdot 10^{4}$	$S_{ua}\cdot 10^{4}$
dual	127.3	138.6	102.2	167.7
lsm	158.7	242.3	281.5	105.7
ef	161.2	198.7	506.1	247.5
adf	134.3	130.2	53.6	115.7

The best performance according to the S_{e2} criterion was given by the *dual* controller, while using the S_{ea} criterion leads to the *adf* controller having the best performance. The accuracy of the *dual* and the *adf* controllers is significantly better then the accuracy obtained by the *lsm* or *ef* controllers. This result is valid for both S_{e2} and S_{ea} criteria. The best results according to the S_{u2} criterion were obtained with the *adf* controller, while the *lsm* controller gave the lowest value of the S_{ua} criterion. Despite this result, the *lsm* controller is not suitable for control of the DR300 system because of the unsatisfactory accuracy of the control process. Higher values of the S_{u2} and the S_{ua} criterions for the *dual* controller comparing to the *adf* controller are causer by excitation of the controlled system incorporated in the bicriterial dual approach. This excitation leads to better identification of the controlled system and subsequently to lower value of the S_{e2} criterion for the *dual* controller.

4 Conclusion

Dual control using the bicriterial approach was verified and compared with some other standard adaptive control approaches in real-time conditions by controlling a laboratory model. Examples of control of a highly non-linear system with a dead zone – the DR300 Speed Control – were shown. The primary aim of this work was not to control the DR300 system but to use it as a demonstration example of the more general class of non-linear systems. Even though the non-linear system was modelled by a linear model, real-time experiments demonstrated very good performance of the dual controller. It should be emphasized that initial parameter estimates were set without using *a priori* information about the controlled system. Usage of converged parameter estimates as an initial setting of the next control process would lead to better performance in the initial phase of the control courses.

Dual bicriterial control is a suitable and promising approach for the control of non-linear systems, time-varying systems, or systems with unknown parameters.

Acknowledgment:
This work was supported by the Operational Programme Research and Development for Innovations co-funded by the Research Development Fund (ERDF) and national budget of Czech Republic within framework of the Centre of Polymer Systems project (reg. number: CZ. 1.05/2.1.00/03.0111).

References:

[1] K. J. Åström and B. Wittenmark, *Adaptive Control*, Massachusetts, Addison-Wesley Publishing Company, 1995.

[2] R. Isermann, K. Lachmann and D. Matko, *Adaptive Control Systems,* Prentice Hall, Inc., Englewood Clils, New Jersey, 1991.

[3] P. E. Welstead and M. B. Zarrop, *Self-tuning Systems: Control and Signal Processing,* John Wiley, Chichester, 1991.

[4] D. Landau, R. Lozano and M. M'Saad, *Adaptive Control,* Springer-Verlag, London, 1998.

[5] V. Bobál, J. Böhm J. Fessl and J. Macháček, *Digital Self-tuning Controllers: Algorithms, Implementation and Applications,* Springer-Verlag, London, 2005.

[6] L. Pekar, A ring for Description and Control of Time-Delay Systems, *WSEAS Transactions on Systems*, Vol. 11, No. 4, 2012, pp. 571-585.

[7] P. Dostál, V. Bobál and Z. Babík, Control of Unstable and Integrating Time Delay Systems, *WSEAS Transactions on Systems*, Vol. 11, No. 4, 2012, pp. 586-595.

[8] M. A. Pakzad, Kalman Filter Design for Time Delay Systems, *WSEAS Transactions on Systems*, Vol. 11, No. 4, 2012, pp. 551-560.

[9] M. A. Pakzad and S. Pakzad, Stability Map of Fractional Order Time-Delay Systems, *WSEAS Transactions on Systems*, Vol. 11, No. 4, 2012, pp. 541-550.

[10] V. Bobál, P. Chalupa, P. Dostál and M. Kubalčík, Design and Simulation Verification of Self-tuning Smith Predictors, *International Journal of Mathematics and Computers in Simulation*, Vol. 5, No. 4, 2011, pp. 342-351.

[11] V. Bobál, P. Chalupa, M. Kubalčík and P. Dostál, Identification and Self-tuning Control of Time-delay Systems, *WSEAS Transactions on Systems*, Vol. 11, No. 4, 2012, pp. 598-606.

[12] R. Prokop, J. Korbel and R. Matušů, Auto-tuning Principles for Time-delay Systems, *WSEAS Transactions on Systems*, Vol. 11, No. 4, 2012, pp. 561-570.

[13] M. Kubalčík and V. Bobál, Predictive Control of Higher Order Systems Approximated by Lower Order Time-delay Models, *WSEAS Transactions on Systems*, Vol. 11, No. 4, 2012, pp. 607-616.

[14] V. Bobál, M. Kubalčík, P. Dostál and J. Matějíček, Adaptive Predictive Control of Time-delay systems, *Computers and Mathematics with Applications*, Vol. 66, 2013, pp. 165-176.

[15] A. A. Feldbaum, Dual Control Theory, *Automation and Remote Control*, Vol. 21, No. I-II, 1960, pp. 1–12, 109–121, Vol. 22, No. III-IV, 1961, pp. 874–880, 1033–1039.

[16] A. A. Feldbaum, *Optimal Control System*, Academic Press, New York, 1965.

[17] N. M. Filatov and H. Unbehauen, Survey of Adaptive Dual Control Methods, *IEE Proceedings Control Theory and Applications*, Vol. 147, No. 1, 2000, pp. 118–122.

[18] B. Lindoff B, J. Holst and B. Wittenmark, Analysis of Approximations of Dual Control, *International Journal of Adaptive Control and Signal Processing*, Vol. 13, No. 7, 1999, pp. 593–620.

[19] B. Wittenmark, Adaptive Dual Control Methods: an Overview, In: *Proceedings of the 5th IFAC Symposium on Adaptive Systems in*

Control and Signal Processing, Budapest, 1995, pp. 67–72.

[20] B. Wittenmark, Adaptive Dual Control, *UNESCO Encyclopedia of Life Support Systems, 6.43.15.6 EOLSS Publishers*, Oxford, UK, http://www.eolss.net/, 2003.

[21] B. Lindoff and J. Holst, Suboptimal Dual Control of Stochastic Systems with Time-Varying Parameters. *Technical reports TFSM-3114*, Lund University of Technology, Sweden, 1997.

[22] A. L. Maitelli and T. A. Yoneyama, A Two-Stage Dual Suboptimal Controller for stochastic systems using approximate moments, *Automatica*, Vol. 30**,** No. 12, 1994, pp. 1949–1954.

[23] A. L. Maitelli and T. Yoneyama, A Multistage Suboptimal Dual Controller Using Optimal Predictors, *IEEE Transactions on Automatic Control*, Vol. 44**,** No. 5, 1999, pp. 1002–1008.

[24] B. Wittenmark and C. Elewitch, An Adaptive Control Algorithm with Dual Features, In: *Preprints of the 7th IFAC/IFORS Symposium on Identification and System Parameter Estimation*, York, UK, 1985.

[25] R. Murray-Smith and D. Sbarbaro, Nonlinear Adaptive Control Using Nonparametric Gaussian Process Prior Models, In: Proceedings of the 15th Triennial World Congress of the International Federation of Automatic Control, Barcelona, Spain, 2002.

[26] M. Šimandl and L. Král, Neural Adaptive Dual Controller with Dynamic Structure, In: *Proceedings of the 7th Portuguese Conference on Automatic Control CONTROLO 2006*, Lisboa, Portugal, 2006.

[27] R. N. da Silva, N. Filatov, J. M. Lemos and H. Unbehauen, A Dual Approach to Start-Up of an Adaptive Predictive Controller, *IEEE Trans. on Control Systems Technology*, Vol. 13, No. 6, 2005, pp. 877-883.

[28] R. Tenno, Dual Adaptive Controls for Linear Systems with Unknown Constant Parameters, *International Journal of Control*, Vol. 83, No. 11, 2010, pp. 2232-2240.

[29] N. M. Filatov and B. Unbehauen, *Adaptive Dual Control. Theory and Application*, Springer-Verlag, Berlin Heidelberg New York, 2004.

[30] N. M. Filatov, H. Unbehauen and U. Keuchel, Dual Pole-placement Controller with Direct Adaptation, *Automatica*, Vol. 33, No. 1, 1997 pp. 113–117.

[31] T. Knohl and H. Unbehauen, Indirect Adaptive Dual Control for Hammerstein Systems Using ANN, *Control Engineering Practice*, Vol. 11, No. 4, 2003, pp. 377–385.

[32] F. Neri, Agent Based Modeling Under Partial and Full Knowledge Learning Settings to Simulate Financial Markets, *AI Communications*, Vol. 25, No. 4, 2012, pp. 295-305.

[33] J. J. Y. Yingying and F. Yanxian, Optimal Sliding-mode Control Scheme for the Position Tracking Servo System, *WSEAS Transactions on Systems*, Vol. 7, No. 5, 2008, pp. 435-444.

[34] Z. Yaou, Z. Wansheng and W. Hongyu, The Study of Recursive-based Sliding Mode Adaptive Control of the Permanent Magnet Synchronous Motor, *WSEAS Transactions on Systems*, Vol. 9, No. 5, 2010, pp. 559-569.

[35] A. M. Yousef, Model Predictive Control Approach Based Load Frequency Controller, *WSEAS Transactions on Systems and Control*, Vol. 6, No. 7, 2011, pp. 265-275.

[36] A. M. Yousef, Experimental Set up Verification of Servo DC Motor Position Control Based on Integral Sliding Mode Approach, *WSEAS Transactions on Systems and Control*, Vol. 7, No. 3, 2012, pp. 87-96.

[37] V. Bobál, P. Chalupa, P. Dostál and J. Novák, Disturbance Rejection of Nonlinear Servo System by Self-tuning Control, *International Journal of Mathematical Models and Methods in Applied Sciences*, Vol. 5, No. 8, 2011, pp. 1431-1438.

[38] V. Bobál and P. Chalupa, *Self-tuning Controllers Simulink Library*. http://www.fai.utb.cz/people/chalupa/stctool/index.htm, 2008.

[39] P. Chalupa, Discrete Decentralized Control Systems, *Ph.D. Thesis*, Tomas Bata University in Zlín, Czech Republic, 2003.

[40] L. Ljung, *System Identification: Theory for the User* (2nd ed.), Prentice-Hall, Englewood Cliffs, New Jersey, 1999.

[41] V. Peterka, A Square-root Filter for Real-time Multivariate Regression, *Kybernetika*, Vol. 11, No. 1, 1975, pp. 53–67.

[42] R. Kulhavý, Restricted Exponential Forgetting in Real Time Identification, *Automatica*, Vol. 23, No. 5, 1987, pp. 586–600.

[43] R. Kulhavý and M. B. Zarrop, On General Concept of Forgetting, *International Journal of Control*, Vol. 58, No. 4, 1993, pp 905–924.

[44] M. S. Radenkovic, Convergence of the Generalised Dual Control Algorithm, *International Journal of Control*, 1988, Vol. 47, No. 5, 1988, pp. 1419–1441.