Control of Unstable and Integrating Time Delay Systems Using Time Delay Approximations

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Abstract: - The paper deals with design of controllers for time delay systems having integrative or unstable properties. The proposed method is based on two methods of time delay approximations. The control system with two feedback controllers is considered. For design of controllers, the polynomial approach is used. Resulting continuous-time controllers obtained via polynomial equations and the LQ control technique ensure asymptotic tracking of step references as well as step disturbances attenuation. Simulation results are presented to illustrate the proposed method.

Key-Words: - Time delay system, Time delay approximation, Polynomial method, LQ control.

1 Introduction

Different classes of technological processes include a time delay in their input-output relations. Plants with a time delay cannot often be controlled by conventional controllers designed without consideration of the dead-time. The control responses using such controllers are often of a poor quality or even can tend to destabilize the closed-loop system.

A part of time delay processes can be unstable or having integrating properties. Typical examples of such processes are e.g. pumps, liquid storing tanks, distillation columns and some types of chemical or biochemical reactors. A control of such processes represents a difficult problem especially for processes containing also other stable or unstable parts with the integrative term.

For control design of unstable and also integrating processes several ways exist. Some methods are based on several modifications of the Smith predictor which was originally developed for stable time delay systems. Such modified Smith predictors were published e.g. in [1] – [4]. Other group of methods employ PID control strategies [5] – [8], the robust control methods [9] and [10] or methods based on the ring of quasipolynomials, e.g. [11]. A solution of differential equations describing the time delay systems can be found e.g. in [12]. Other simulation possibilities are described e.g. in [13].

This paper presents one method of the controller design for unstable and integrating time delay systems and also for its combination with a stable or an unstable first order system. The presented procedure is based on approximations of the time delay term by the first order Taylor numerator expansion (TNE) and by the first order Padé approximation (PA). The control system with two feedback controllers is considered, see, e.g. [14], [15]. The controllers are derived using the polynomial approach published e.g. in [16]. For tuning of controller parameters, the pole assignment method exploiting the LQ control technique is used, see, e.g. [17]. The resulting proper and stable controllers obtained via polynomial Diophantine equations and spectral factorization techniques ensure the asymptotic tracking of step references as well as step disturbances attenuation.

The structures of developed controllers together with analytically derived formulas for computation of their parameters are presented for five typical plants of time delay systems: the unstable first order time delay system (UFOTDS), the unstable second order time delay system (USOTDS), integrating time delay system (ITDS), and, the stable and unstable first order plus integrating time delay system (SFOPITDS, UFOPITDS).

Presented simulation results obtained by both approximations document usefulness of the proposed method providing stable control responses of a good quality.

2 Approximate Transfer Functions

The transfer functions in the sequence UFOTDS, USOTDS, ITDS, SFOPITDS and UFOPITDS have forms

$$G_1(s) = \frac{K}{\tau s - 1} e^{-\tau_d s} \tag{1}$$

$$G_2(s) = \frac{K}{(\tau_1 s - 1)(\tau_2 s + 1)} e^{-\tau_d s} \tag{2}$$

$$G_3(s) = \frac{K}{s} e^{-\tau_d s} \tag{3}$$

$$G_4(s) = \frac{K}{s(\tau s \pm 1)} e^{-\tau_d s} \tag{4}$$

2.1 TN expansion

In the first case, the time delay terms in (1) – (4) are approximated by the TN expansion

$$e^{-\tau_d s} \approx 1 - \tau_d s \,. \tag{5}$$

Then, approximate transfer functions relating to (1) – (4) have forms

$$G_{1N}(s) = \frac{K(1-\tau_d s)}{\tau s - 1} = \frac{b_0 - b_1 s}{s - a_0} \tag{6}$$

where

$$b_0 = \frac{K}{\tau},\; b_1 = \frac{K\tau_d}{\tau},\; a_0 = \frac{1}{\tau} \tag{7}$$

for the UFOTDS,

$$G_{2N}(s) = \frac{K(1-\tau_d s)}{(\tau_1 s - 1)(\tau_2 s + 1)} = \frac{b_0 - b_1 s}{s^2 + a_1 s + a_0} \tag{8}$$

where

$$b_0 = \frac{K}{\tau_1 \tau_2},\; b_1 = \frac{K\tau_d}{\tau_1 \tau_2},\; a_0 = \frac{1}{\tau_1 \tau_2},\; a_1 = \frac{\tau_1 - \tau_2}{\tau_1 \tau_2} \tag{9}$$

for the USOTDS,

$$G_{3N}(s) = \frac{K(1-\tau_d s)}{s} = \frac{b_0 - b_1 s}{s} \tag{10}$$

where

$$b_0 = K,\;\; b_1 = K\tau_d \tag{11}$$

for the ITDS, and,

$$G_{4,5N}(s) = \frac{K(1-\tau_d s)}{s(\tau s \pm 1)} = \frac{b_0 - b_1 s}{s^2 + a_1 s} \tag{12}$$

where

$$b_0 = \frac{K}{\tau},\;\; b_1 = \frac{K\tau_d}{\tau},\;\; a_1 = \pm\frac{1}{\tau} \tag{13}$$

for the SFOPITDS and UFOPITDS.

2.2 Padé approximation

In the second case, the time delay terms in (1) – (4) are approximated by the by the first order Padé approximation

$$e^{-\tau_d s} \approx \frac{2 - \tau_d s}{2 + \tau_d s} \tag{14}$$

Now, approximate transfer functions in the same sequence take forms

$$G_{1P}(s) = \frac{K(2-\tau_d s)}{(\tau s - 1)(2 + \tau_d s)} = \frac{b_0 - b_1 s}{s^2 + a_1 s - a_0} \tag{15}$$

where

$$b_0 = \frac{2K}{\tau\tau_d},\; b_1 = \frac{K}{\tau},\; a_0 = \frac{2}{\tau\tau_d},\; a_1 = \frac{2\tau - \tau_d}{\tau\tau_d} \tag{16}$$

for the UFOTDS,

$$G_{2P}(s) = \frac{K(2-\tau_d s)}{(\tau_1 s - 1)(\tau_2 s + 1)(2 + \tau_d s)} = \frac{b_0 - b_1 s}{s^3 + a_2 s^2 + a_1 s - a_0} \tag{17}$$

where

$$b_0 = \frac{2K}{\tau_1 \tau_2 \tau_d},\; b_1 = \frac{K}{\tau_1 \tau_2},\; a_0 = \frac{2}{\tau_1 \tau_2 \tau_d}$$
$$a_1 = \frac{2(\tau_1 - \tau_2) - \tau_d}{\tau_1 \tau_2 \tau_d},\; a_2 = \frac{2\tau_1\tau_2 + \tau_1\tau_d - \tau_2\tau_d}{\tau_1 \tau_2 \tau_d} \tag{18}$$

for the USOTDS,

$$G_{3P}(s) = \frac{K(2-\tau_d s)}{s(2 + \tau_d s)} = \frac{b_0 - b_1 s}{s^2 + a_1 s} \tag{19}$$

where

$$b_0 = \frac{2K}{\tau_d},\; b_1 = K,\; a_1 = \frac{2}{\tau_d} \tag{20}$$

for the ITDS, and,

$$G_{4,5P}(s) = \frac{K(2-\tau_d s)}{s(\tau s \pm 1)(2 + \tau_d s)} = \frac{b_0 - b_1 s}{s^3 + a_2 s^2 + a_1 s} \tag{21}$$

where

$$b_0 = \frac{2K}{\tau\tau_d},\ b_1 = \frac{K}{\tau},\ a_1 = \pm\frac{2}{\tau\tau_d},\ a_2 = \frac{2\tau \pm \tau_d}{\tau\tau_d} \quad (22)$$

for the SFOPITDS and UFOPTDS.

Remark: For the UFOTDS and the UFOPITDS the conditions $\tau_d \neq \tau$ in (6) and (12), $\tau_d \neq \tau_1$ in (8), and, $\tau_d \neq 2\tau$ in (15) and (21) must be fulfilled.

All approximate transfer functions have the form

$$G_A(s) = \frac{b(s)}{a(s)} \quad (23)$$

where b and a are coprime polynomials in s that fulfill the inequality $\deg b \leq \deg a$.

3 Control Design

The control system with two feedback controllers is depicted in Fig.1.

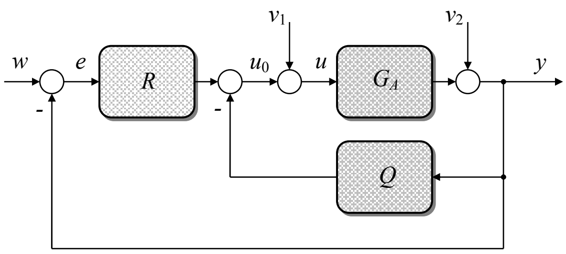

Fig.1. Control system.

In the scheme, w is the reference signal, v_1, v_2 are input and output disturbances, e is the tracking error, u_0 is the controller output, y is the controlled output and u is the control input. The reference w and both disturbances v_1 and v_2 are considered to be step functions with transforms

$$W(s) = \frac{w_0}{s},\ V_1(s) = \frac{v_{10}}{s},\ V_2(s) = \frac{v_{20}}{s} \quad (24)$$

The transfer function G_A represents a proper approximate transfer function in the general form (23).

The transfer functions of controllers are

$$Q(s) = \frac{\tilde{q}(s)}{\tilde{p}(s)},\ R(s) = \frac{r(s)}{\tilde{p}(s)} \quad (25)$$

where $\tilde{q}, r$ and $\tilde{p}$ are coprime polynomials in s.

3.1 Application of Polynomial Method

The controller design described in this section follows from the polynomial approach. The general conditions required to govern the control system properties are formulated as follows:

- Strong stability of the control system (in addition to the control system stability, also the stability of a controller is required).
- Internal properness of the control system.
- Asymptotic tracking of the reference.
- Attenuation of disturbances.

The procedure to derive admissible controllers can be carried out as follows:

Transforms of the controlled output and the tracking error take the form (for simplification, the argument s is in some equations omitted)

$$Y(s) = \frac{1}{d}\left[br\,W(s) + b\tilde{p}\,V_1(s) + a\tilde{p}V_2(s)\right] \quad (26)$$

$$E(s) = \frac{1}{d}\left[(a\tilde{p} + b\tilde{q})W(s) - b\tilde{p}V_1(s) - a\tilde{p}V_2(s)\right] \quad (27)$$

where

$$d(s) = a(s)\,\tilde{p}(s) + b(s)\left(r(s) + \tilde{q}(s)\right) \quad (28)$$

is the characteristic polynomial with roots as poles of the closed-loop.

Establishing the polynomial t as

$$t(s) = r(s) + \tilde{q}(s) \quad (29)$$

and substituting (29) into (28), the condition of the control system stability is ensured when polynomials $\tilde{p}$ and t are given by a solution of the polynomial Diophantine equation

$$a(s)\tilde{p}(s) + b(s)t(s) = d(s) \quad (30)$$

with a stable polynomial d on the right side.

With regard to (24), asymptotic tracking and both disturbances attenuation are provided by divisibility of both terms $a\tilde{p} + b\tilde{q}$ and $\tilde{p}$ in (27) by s. This condition is fulfilled for polynomials $\tilde{p}$ and $\tilde{q}$ in the form

$$\tilde{p}(s) = s\,p(s),\ \tilde{q}(s) = s\,q(s). \quad (31)$$

Subsequently, the transfer functions of controllers take forms

$$Q(s) = \frac{q(s)}{p(s)},\ R(s) = \frac{r(s)}{s\,p(s)}. \quad (32)$$

A stable polynomial $p(s)$ in denominators of (32) ensures the stability of ontrollers.

The control system satisfies the condition of internal properness when the transfer functions of all its components are proper. Consequently, the degrees of polynomials q and r must fulfill inequalities

$$\deg q \le \deg p\,,\quad \deg r \le \deg p + 1\,. \tag{33}$$

Now, the polynomial t can be rewritten to the form

$$t(s) = r(s) + s\,q(s)\,. \tag{34}$$

Taking into account solvability of (30) and conditions (33), the degrees of polynomials in (30) and (32) can be easily derived as

$$\begin{gathered}\deg t = \deg r = \deg a\,,\ \deg q = \deg a - 1\\ \deg p = \deg a - 1\,,\ \ \deg d = 2\deg a\,.\end{gathered} \tag{35}$$

Denoting deg $a = n$, polynomials t, r and q have the form

$$t(s) = \sum_{i=0}^{n} t_i s^i\,,\ r(s) = \sum_{i=0}^{n} r_i s^i\,,\ q(s) = \sum_{i=1}^{n} q_i s^{i-1} \tag{36}$$

and among of their coefficients equalities

$$r_0 = t_0\,,\ \ r_i + q_i = t_i \ \text{ for } i = 1, \dots, n \tag{37}$$

hold. Since by a solution of the polynomial equation (30) only coefficients t_i can be calculated, unknown coefficients r_i and q_i can be obtained by a choice of selectable coefficients $\beta_i \in \langle 0,1 \rangle$ such that

$$r_i = \beta_i t_i\,,\ \ q_i = (1-\beta_i)t_i \ \text{ for } i = 1, \dots, n\,. \tag{38}$$

The coefficients β_i distribute a weight between numerators of transfer functions Q and R. With respect to the transform (26), it may be expected that higher values of β_i speed up control responses to step references.

Remark: If $\beta_i = 1$ for all i, the control system in Fig. 1 demotes to the 1DOF control configuration. If $\beta_i = 0$ for all i and the reference and both disturbances are step functions, the control system corresponds to the 2DOF control configuration.

The controller parameters then follow from solutions of the polynomial equation (30) and depend upon coefficients of polynomial d. The next problem here means to find a stable polynomial d that enables to obtain the acceptable stabilizing and stable controllers.

3.2 Pole Assignment

In this paper, the polynomial d is considered as a product of two stable polynomials g and m in the form

$$d(s) = g(s)\,m(s) \tag{39}$$

where the polynomial g is a monic form of the polynomial h obtained by spectral factorization

$$[s\,a(s)]^* \varphi [s\,a(s)] + b^*(s)b(s) = h^*(s)\,h(s) \tag{40}$$

where $\varphi > 0$ is the weighting coefficient.

Remark: In the LQ control theory, the spectral factorization (40) is used in a procedure of minimization of the quadratic cost function

$$J = \int_0^\infty \left\{ e^2(t) + \varphi \dot{u}^2(t) \right\} dt \tag{41}$$

where $e(t)$ is the tracking error and $\dot{u}(t)$ is the control input derivative.

The polynomials h and derived formulas for their parameters calculation have forms

$$h(s) = h_2 s^2 + h_1 s + h_0 \tag{42}$$

for the UFOTDS and ITDS with the TN expansion where

$$h_0 = |b_0|\,,\ \ h_2 = \sqrt{\varphi}\,,\ \ h_1 = \sqrt{\varphi a_0^2 + b_1^2 + 2h_0 h_2} \tag{43}$$

and $a_0 = 0$ for the ITDS,

$$h(s) = h_3 s^3 + h_2 s^2 + h_1 s + h_0 \tag{44}$$

for the UFOTDS and ITDS with the Padé approximation, and, for the USOTDS, SFOPITDS and UFOPITDS with the TN expansion where

$$\begin{gathered} h_0 = |b_0|, h_3 = \sqrt{\varphi}, h_1 = \sqrt{\varphi a_0^2 + b_1^2 + 2h_0 h_2}\\ h_2 = \sqrt{\varphi(a_1^2 - 2a_0) + 2h_1 h_3}\end{gathered} \tag{45}$$

and $a_0 = 0$ for the ITDS, SFOPITDS and UFOPITDS, and,

$$h(s) = h_4 s^4 + h_3 s^3 + h_2 s^2 + h_1 s + h_0 \tag{46}$$

for the USOTDS, SFOPITDS and UFOPITDS with the Padé approximation where

$$\begin{gathered} h_0 = |b_0|, h_4 = \sqrt{\varphi}, h_1 = \sqrt{\varphi a_0^2 + b_1^2 + 2h_0 h_2}\\ h_2 = \sqrt{\varphi(a_1^2 - 2a_0 a_2) + 2h_1 h_3 - 2h_0 h_4}\\ h_3 = \sqrt{\varphi(a_2^2 - 2a_1) + 2h_2 h_4}\end{gathered} \tag{47}$$

and $a_0 = 0$ for both SFOPITDS and UFOPITDS. For calculation of d, polynomials (42), (44) and (46) are arranged to monic forms $g(s)$ (with unit coefficients by the highest power of s) such that

$$g_j = h_j / h_n \quad j = 0,1,\dots,n \tag{48}$$

where $n = \deg h$.

The second polynomial m ensuring properness of the controller is chosen as

$$m(s) = 1 \tag{49}$$

for both UFOTDS and ITDS with the TN expansion,

$$m(s) = s + \frac{2}{\tau_d} \tag{50}$$

for both UFOTDS and ITDS with the Padé approximation,

$$m(s) = s + \frac{1}{\tau_2} \tag{51}$$

for the USOTDS with the TN expansion,

$$m(s) = \left(s + \frac{1}{\tau_2}\right)\left(s + \frac{2}{\tau_d}\right) \tag{52}$$

for the USOTDS with the Padé approximation,

$$m(s) = s + \frac{1}{\tau} \tag{53}$$

for both SFOPITDS and UFOPITDS with the TN expansion, and,

$$m(s) = \left(s + \frac{2}{\tau_d}\right)\left(s + \frac{1}{\tau}\right) \tag{54}$$

for both UFOPITDS and SFOPITDS with the Padé approximation.

The above forms of m lead to the polynomial d with coefficients containing only the selectable parameter φ with all other coefficients depending on parameters of polynomials b and a. Consequently, a location of the closed loop poles can be affected by the selectable parameter φ.

The transfer functions of controllers with degrees of polynomials in their numerators and denominators given by (35) are

$$Q(s) = \frac{q_1}{p_0}, \quad R(s) = \frac{r_1 s + r_0}{p_0 s} \tag{55}$$

for both UFOTDS and ITDS with the TN expansion,

$$Q(s) = \frac{q_2 s + q_1}{s + p_0}, \quad R(s) = \frac{r_2 s^2 + r_1 s + r_0}{s(s + p_0)} \tag{56}$$

for both UFOTDS and ITDS with the Padé approximation, and, for the USOTDS, SFOPITDS and UFOPITDS with the TN expansion. Further,

$$Q(s) = \frac{q_3 s^2 + q_2 s + q_1}{s^2 + p_1 s + p_0}$$
$$R(s) = \frac{r_3 s^3 + r_2 s^2 + r_1 s + r_0}{s(s^2 + p_1 s + p_0)} \tag{57}$$

for the USOTDS, SFOPITDS and UFOPITDS with the Padé approximation.

In all cases, the parameters q in numerators of controllers are computed from parameters t according to (37).

For clarity, derived formulas for computation of parameters p_0 and t the controller derived for all considered cases together with conditions of the controllers' stability are introduced in the form of tables.

Table 1. Controller parameters for UFOTDS

TN expansion
$p_0 = \dfrac{\tau\tau_d (g_1 + \tau_d g_0) + \tau}{\tau - \tau_d}$ $t_0 = \dfrac{\tau}{K} g_0, \quad t_1 = \dfrac{1}{K}\dfrac{\tau}{\tau_d}(p_0 - 1)$
$p_0 > 0$ for $\tau_d < \tau$
Padé approximation
$p_0 = \dfrac{\tau\left[2g_2 + \tau_d (g_1 + \frac{\tau_d}{2} g_0)\right] + 2}{2\tau - \tau_d}, \quad t_0 = \dfrac{\tau}{K} g_0$ $t_1 = \dfrac{1}{K}[p_0 + \tau(g_1 + \tau_d g_0)]$, $t_2 = \dfrac{1}{K}[\tau(p_0 - g_2) - 1]$
$p_0 > 0$ for $\tau_d < 2\tau$

Table 2. Controller parameters for ITDS

TN expansion
$p_0 = 1 + \tau_d (g_1 + K\tau_d)$ $t_0 = r_0 = \dfrac{1}{K} g_0, \quad t_1 = \dfrac{1}{K}(g_1 + \tau_d g_0)$
$p_0 > 0$ for all τ_d
Padé approximation

$p_0 = g_2 + \frac{\tau_d}{4}(2g_1 + \tau_d g_0)\,,\;\; t_0 = r_0 = \frac{1}{K} g_0$ $t_1 = \frac{1}{K}(g_1 + \tau_d g_0)\,,\;\; t_2 = \frac{\tau_d}{4K}(2g_1 + \tau_d g_0)$
$p_0 > 0$ for all τ_d

Table 3. Controller parameters for USOTDS

TN expansion
$p_0 = \frac{\tau_1(g_2 + \tau_d g_1 + \tau_d^2 g_0) + 1}{\tau_1 - \tau_d}$ $t_0 = \frac{\tau_1}{K} g_0\,,\;\; t_1 = \frac{1}{K}\left[p_0 + \tau_1 g_1 + \tau_1(\tau_2 + \tau_d) g_0\right]$ $t_2 = \frac{1}{K}\frac{\tau_1 \tau_2}{\tau_d}\left[p_0 - g_2 - \frac{1}{\tau_1}\right]$
$p_0 > 0$ for $\tau_d < \tau_1$
Padé approximation
$p_0 = \frac{2g_3 + \tau_1\left[2g_2 + \tau_d\left(g_1 + \frac{\tau_d}{2} g_0\right)\right] + \frac{2}{\tau_1}}{2\tau_1 - \tau_d}$ $p_1 = g_3 + \frac{1}{\tau_1}$ $t_0 = \frac{\tau_1}{K} g_0\,,\; t_1 = \frac{1}{K}\left[p_0 + \tau_1\left(g_1 + (\tau_2 + \tau_d) g_0\right)\right]$ $t_2 = \frac{1}{K}\left[\left(\frac{4\tau_1\tau_2}{\tau_d} + \tau_1 - \tau_2\right) p_0 - \right.$ $\left. -\left(\frac{4\tau_2}{\tau_d} + 1\right)\left(g_3 + \tau_1 g_2 + \frac{1}{\tau_1}\right) - \tau_1 \tau_2 g_1\right]$ $t_3 = \frac{\tau_2}{K}\left[\tau_1(p_0 - g_2) - g_3 - \frac{1}{\tau_1}\right]$
$p_0 > 0$ for $\tau_d < 2\tau_1$

Table 4. Controller parameters for SFOPITDS

TN expansion
$p_0 = g_2 + \tau_d(g_1 + \tau_d g_0)\,,\;\; t_0 = \frac{1}{K} g_0$ $t_1 = \frac{1}{K}[g_1 + (\tau + \tau_d) g_0]\,,\;\; t_2 = \frac{\tau}{K}(g_1 + \tau_d g_0)$
$p_0 > 0$ for all τ_d
Padé approximation
$p_0 = g_2 + \frac{\tau_d}{4}(2g_1 + \tau_d g_0)\,,\;\; p_1 = g_3$ $t_0 = \frac{1}{K} g_0\,,\; t_1 = \frac{1}{K}[g_1 + (\tau + \tau_d) g_0]$ $t_2 = \frac{1}{4K}[(2\tau + \tau_d)(2g_1 + \tau_d g_0) + 2\tau\tau_d g_0]$ $t_3 = \frac{\tau\tau_d}{4K}(2g_1 + \tau_d g_0)$
$p_1 > 0$ for all τ_d, $p_0 > 0$ for all τ_d

Table 5. Controller parameters for UFOPITDS

TN expansion
$p_0 = \frac{(\tau + \tau_d)[g_2 + \tau_d(g_1 + \tau_d g_0)] + 2\tau}{\tau - \tau_d}$
$t_0 = \frac{1}{K} g_0\,,\;\; t_1 = \frac{1}{K}[g_1 + (\tau + \tau_d) g_0]$ $t_2 = \frac{1}{K}\frac{2\tau g_2 + \tau(\tau + \tau_d)(g_1 + \tau_d g_0) + 2}{\tau - \tau_d}$
$p_0 > 0$ for $\tau_d < \tau$
Padé approximation
$p_0 = \frac{4g_3 + (2\tau + \tau_d)\left(g_2 + \frac{\tau_d}{2} g_1 + \frac{\tau_d^2}{4} g_0\right) + \frac{4}{\tau}}{2\tau - \tau_d}$ $p_1 = g_3 + \frac{2}{\tau}$ $t_0 = \frac{1}{K} g_0\,,\; t_1 = \frac{1}{K}[g_1 + (\tau + \tau_d) g_0]$ $t_2 = \frac{1}{K}\left[\left(\frac{4\tau}{\tau_d} - 1\right) p_0 - \frac{8}{\tau_d} g_3 - \left(\frac{4\tau}{\tau_d} + 1\right) g_2 - \right.$ $\left. -\tau g_1 - \frac{8}{\tau\tau_d}\right]$ $t_3 = \frac{1}{K}\left[\tau(p_0 - g_2) - 2g_3 - \frac{2}{\tau}\right]$
$p_1 > 0$ for all τ_d , $p_0 > 0$ for $\tau_d < 2\tau$

4 Simulation Results

All simulations were performed by MATLAB-Simulink tools. In all cases, the unit step reference w was introduced at the time t = 0 and the step disturbances v_1 and v_2 were subsequently injected after settling of the control responses.

4.1 UFOTDS

The parameters in the transfer function (1) has been chosen as $K = 1$ and $\tau = 4$.

The responses in Fig.2 document applicability of the TNE for the UFOTDS with a small value of τ_d. Further, the responses illustrate necessity of a higher value of φ to achieving of an aperiodic character of responses. Smaller values of φ lead to their oscillatory character. An effect of the parameter β_1 can be seen in Fig.3. Its increasing value speeds the control but causes expressive overhoots.

A preference of the PA in comparison with the TN is evident from the controlled output responses in Fig.4 computed under the same conditions. Moreover, the PA enables a use also for higher values of τ_d as shown in Fig.5.

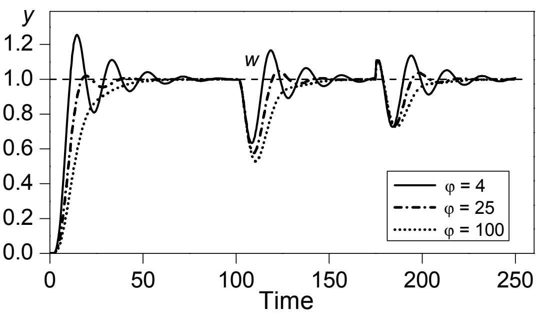

Fig.2. UFOTDS - TNE: Controlled output for various φ ($\tau_d = 2$, $\beta_1 = 0$, $v_1 = -0.2$, $v_2 = 0.1$).

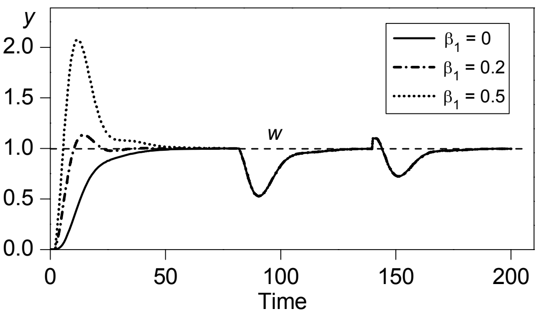

Fig.3. UFOTDS - TNE: Controlled output for various β_1 ($\tau_d = 2$, $\varphi = 100$, $v_1 = -0.2$, $v_2 = 0.1$).

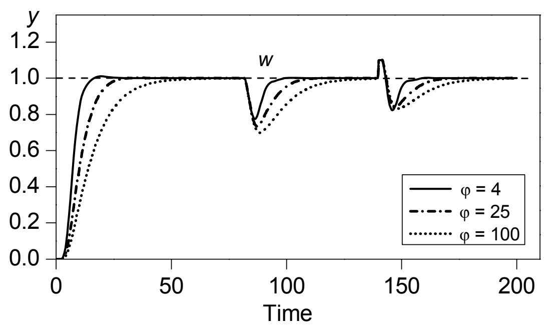

Fig.4. UFOTDS - PA: Controlled output for various φ ($\tau_d = 2$, $\beta_{1,2} = 0$, $v_1 = -0.2$, $v_2 = 0.1$).

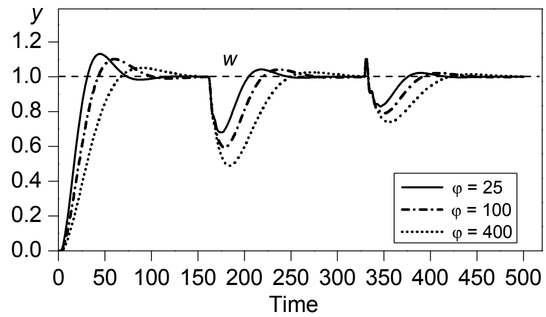

Fig.5. UFOTDS - PA: Controlled output for various φ ($\tau_d = 4$, $\beta_{1,2} = 0$, $v_1 = -0.2$, $v_2 = 0.1$).

4.2 USOTDS

The parameters in the transfer function (2) were chosen as $K = 1$, $\tau_1 = 4$, $\tau_2 = 2$.

Also in this case, an application of the TNE is possible for smaller values of the time delay and for higher values of φ. A higher value of τ_d needs a use of the PA. The simulation results can be seen in Figs.6 and 7.

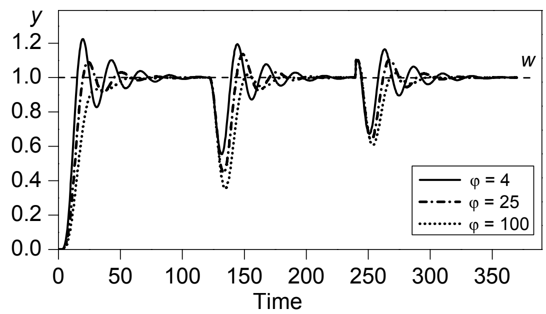

Fig.6. USOTDS - TNE: Controlled output for various φ ($\tau_d = 2$, $\beta_{1,2} = 0$, $v_1 = -0.2$, $v_2 = 0.1$).

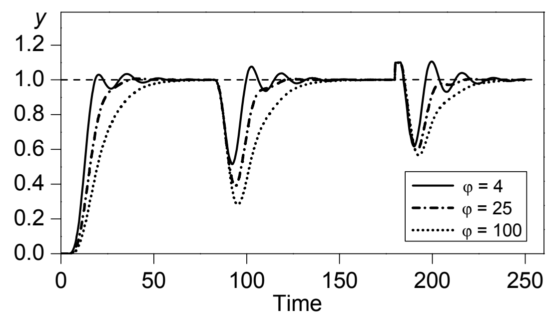

Fig.7. USOTDS - PA: Controlled output for various α ($\tau_d = 3$, $\beta_{1,2,3} = 0$, $v_1 = -0.2$, $v_2 = 0.1$).

The responses in Fig.8 demonstrate their high sensitivity to parameters β. Evidently, on behalf of acceleration of the control, only small values β should be chosen. Their higher values lead to expresive overshoots at the start of the tracking interval.

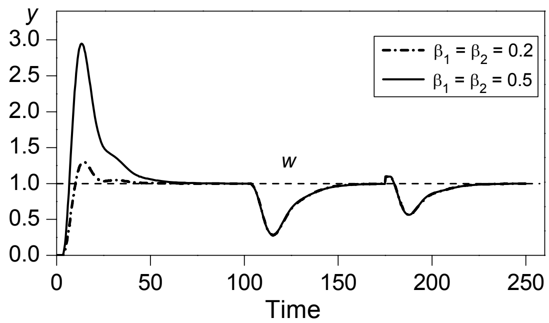

Fig.8. USOTDS - PA: Controlled output for various β_1, β_2 ($\tau_d = 3$, $\varphi = 100$, $\beta_3 = 0$, $v_1 = -0.2$, $v_2 = 0.1$).

4.3 ITDS

In this case, the parameter in (3) has been chosen as $K = 0.2$.

The responses in Fig.9 document applicability of the TNE for the ITDS with smaller values of τ_d. There is not a significant difference in comparison with utilization of the PA as shown in Fig.10. Here, also a selection of the parameter φ is not very important.

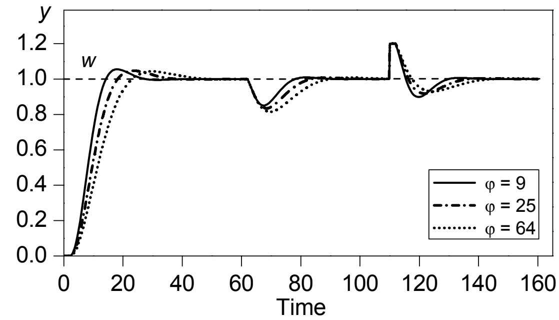

Fig.9. ITDS - TNE: Controlled output for various φ ($\tau_d = 2$, $\beta_1 = 0$, $v_1 = -0.2$, $v_2 = 0.2$).

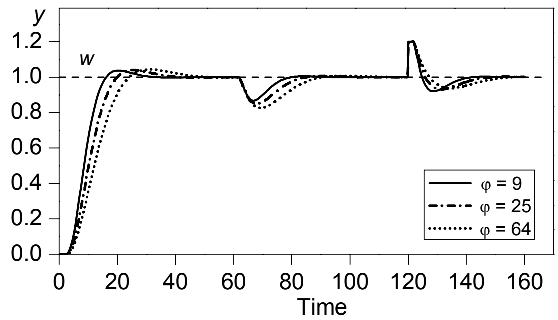

Fig.10. ITDS - PA: Controlled output for various φ ($\tau_d = 2$, $\beta_1 = 0$, $v_1 = -0.2$, $v_2 = 0.2$).

An effect of the parameter β_1 on the controlled output responses can be seen in Fig.11. A reasonable choice of this parameter can accelerate the control responses keeping their apperiodic character.

A difference between both approximations appears for higher values of τ_d as it can be seen in Figs.12, 13 and 14. There, a priority of the PA is evident. It is also clear that a higher value of τ_d requires a use of a higher value of φ.

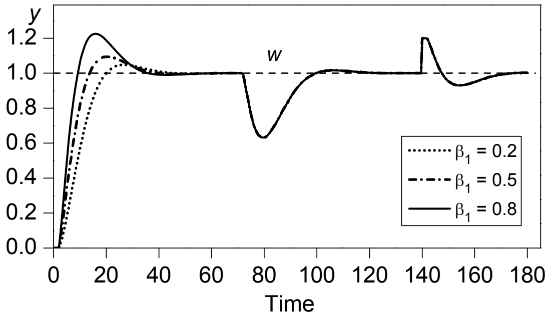

Fig.11. ITDS - TNE: Controlled output for various β_1 ($\tau_d = 5$, $\varphi = 25$, $v_1 = -0.4$, $v_2 = 0.2$).

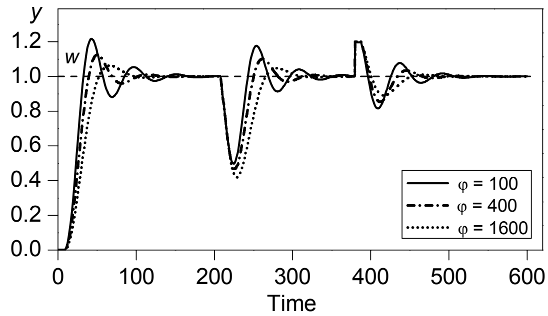

Fig.12. ITDS - TNE: Controlled output for various φ ($\tau_d = 8$, $\beta_1 = 0$, $v_1 = -0.2$, $v_2 = 0.2$).

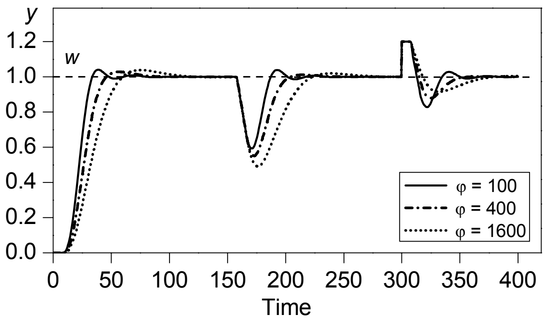

Fig.13. ITDS - PA: Controlled output for various φ ($\tau_d = 8$, $\beta_1 = \beta_2 = 0$, $v_1 = -0.2$, $v_2 = 0.2$).

4.4 SFOPITDS

For this model (and, also for the UFOPITDS), the parameters in (2) have been chosen as $K = 0.2$ and $\tau = 4$. The controlled output responses for various φ are shown in Figs.15 and 16, a comparison between application of the TNE and PA can be seen in Fig.17. The presented results clearly prove a better control quality obtained by the PA. It should be noted that for both SFOPITDS and UFOPITDS zero parameters β were chosen equivalent to the 2DOF control structure. This choice gave best control results.

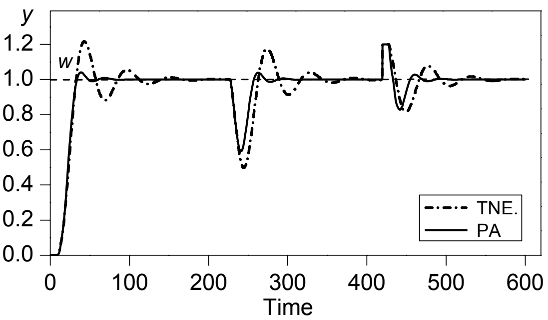

Fig.14. ITDS – Comparison of controlled outputs for TNE and PA ($\tau_d = 8$, $\varphi = 100$, $\beta_1 = \beta_2 = 0$, $v_1 = -0.2$, $v_2 = 0.2$).

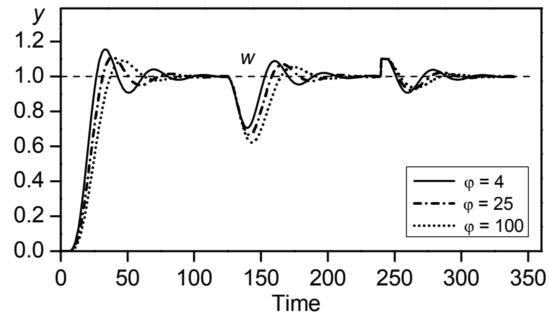

Fig.15. SFOPITDS - TNE: Controlled output for various φ ($\tau_d = 5$, $v_1 = -0.2$, $v_2 = 0.1$).

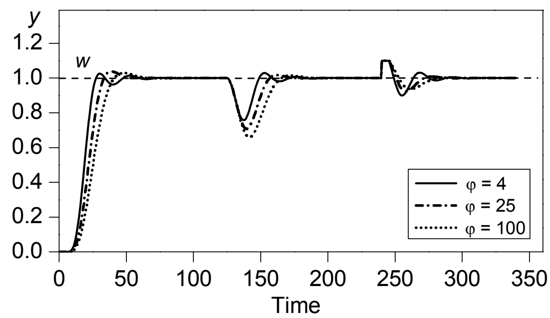

Fig.16. SFOPITDS - PA: Controlled output for various φ ($\tau_d = 5$, $v_1 = -0.1$, $v_2 = 0.1$).

4.5 UFOPITDS

With regard to a presence of both integrating and unstable parts, the UFOPITDSs belong to hardly controllable systems. However, the control responses in Fig.18 document usability of both TNE and PA for smaller value of τ_d. Higher values of τ_d require a selection of higher values of φ as shown for the PA in Fig.19 However, for higher values of φ, the TNE is unsuitable, as documented in Fig.20.

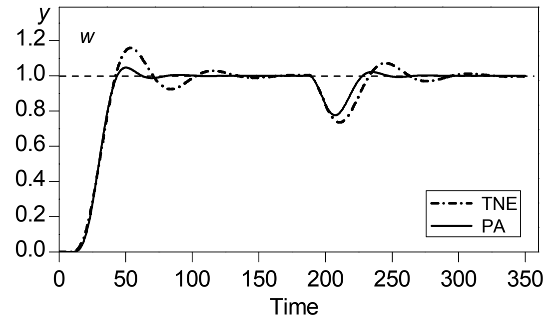

Fig.17. SFOPITDS - Comparison of controlled outputs for TNE and PA ($\tau_d = 8$, $\varphi = 100$, $v_1 = -0.1$, $v_2 = 0.2$)

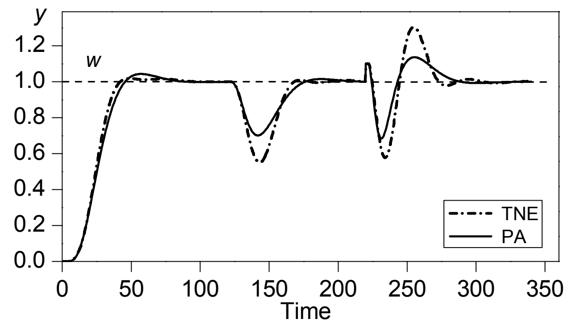

Fig.18. UFOPITDS - Comparison of controlled outputs for TNE and PA ($\tau_d = 2$, $\varphi = 400$, $v_1 = -0.05$, $v_2 = 0.1$).

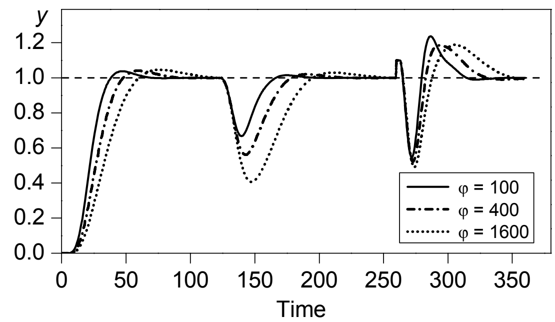

Fig.19. UFOPITDS - PA: Controlled output for various φ ($\tau_d = 3$, $v_1 = -0.05$, $v_2 = 0.1$).

5 Conclusions

The problem of control design for unstable and integrating time delay systems has been solved and analysed. The proposed method is based in two ways of the time delay approximation. The controller design uses the polynomial synthesis and the controller setting employs the results of the LQ control theory. The presented procedure provides satisfactory control responses in the tracking of a step reference as well as in step disturbances attenuation. The presented results have demonstrate the usability of the method and the control of a good quality also for relatively high ratio of the time delay to the time constant. The procedure makes possible a tuning of the controller parameters by two types of selectable parameters. Using derived formulas, the controller parameters can be automatically computed. From this reason, the method could also be used for an adaptive control.

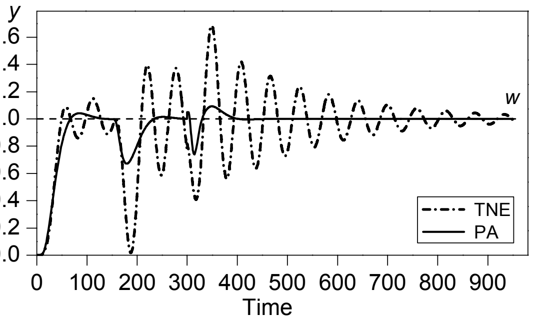

Fig.20. UFOPITDS – Comparison of controlled outputs for TNE and PA ($\tau_d = 3$, $\varphi = 2500$, $v_1 = -0.05$, $v_2 = 0.1$).

Acknowledgments
This article was written in part with support of Operational Program Research and Development for Innovations co-funded by the European Regional Development Fund (ERDF) and national budget of Czech Republic, within the framework of project Centre of Polymer Systems (reg. number: CZ.1.05/2.1.00/03.0111) and with support of the grant IGA/FAI/2012/020.

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