Possible Solution of Decoupling and Invariance of Multi-variable Control Loop by Using Binding and Correction Members

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Abstract: - The paper describes one of possible methods how to control of multi-variable control loops. In this case the used method of control uses the so called main controllers, binding members and correction members. This control method combines classical approach for ensuring decoupling of multi-variable control loop by means of binding members and the use of the correction members for ensuring invariance of multi-variable control loop by means of two approaches. Main controllers can be proposed by arbitrary single-variable synthesis method. Simulation verification of the used control method is carried out for example of three-variable loop of a steam turbine.

Key-Words: - Decoupling of control loop, control, invariance of control loop, simulation.

1 Introduction

It is often required, at large numbers controlled plants, that their input and output variables have to be controlled simultaneously. The examples these controlled plants are e.g. aircraft autopilots, air-conditioning plants, chemical processes, distillation columns, steam boilers, spacecraft, steam turbines, etc. [1]. In these cases, it means that there is not only larger number of independent SISO (single-variable) control loop. These control loops are complex with several controlled variables where separate variables are not mutually independent. Mutual coupling of controlled variables is usually given by simultaneous action of each of input variables of controlled plant (manipulated variables and disturbance variables) to all controlled variables. These control loops are called MIMO (multi-variable) control loops and they are a complex of mutually influencing simpler control loops [1]. Special case of MIMO control loop is SISO control loop that have only one input signal (disturbance variable, manipulated variable) and one output signal (controlled variable) [2], [3].

One of possible examples of MIMO controlled plant is also an above mentioned steam turbine [4]–[6]. In the experimental part of the paper is considered three-variable controlled plant of steam turbine [4]. The selected method of control of the MIMO controlled plant uses the so called main controllers, binding members and correction members [1]. The main controllers can be designed via arbitrary SISO synthesis methods, e.g. [1], [7]–[12]. Binding members are determined from main controllers and from parameters of MIMO controlled plant and ensuring decoupling of MIMO control loop. Correction members are determined from parameters of MIMO controlled plant and ensure an elimination of influence of disturbance variables on MIMO control loop, i.e. the correction members ensure invariance of MIMO control loop. In the next part of the paper are described two approaches for ensuring invariance of MIMO control loop by means of correction members. The control method of MIMO control loop, described in the next part of this paper, is considered for MIMO controlled plant with same number input signals and output signals.

All simulation experiments were performed in the simulation mathematical education and research software MATLAB/SIMULINK [12]. MATLAB is a widely used tool not in education but also in research; in addition to that, many researchers have produced a wide variety of educational tools based on MATLAB [14], [15].

2 Multi-Variable Control Loop

2.1 Description of used multi-variable control loop

It will be considered multi-variable control loop with measurement of disturbance variables via the following figure (see Fig. 1) [1].

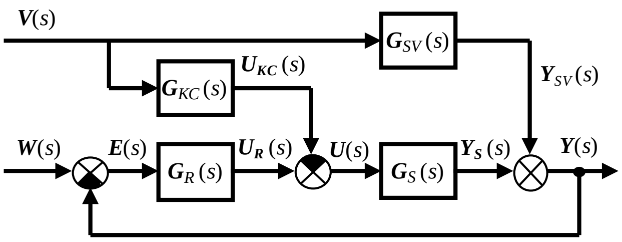

Fig. 1 - Multi-variable control loop with measurement of disturbance variables

The description of the separate parameters in the above mentioned multi-variable control is following, i.e. $\boldsymbol{G}_S(s)$, $\boldsymbol{G}_{SV}(s)$, $\boldsymbol{G}_R(s)$ and $\boldsymbol{G}_{KC}(s)$ are transfer function matrices of a controlled plant, disturbance variables, controller and correction members. Signal $\boldsymbol{Y}(s)$ $[n\times1]$ is a vector of controlled variables, $\boldsymbol{W}(s)$ $[n\times1]$ is a vector of setpoints, $\boldsymbol{U}(s)$ $[n\times1]$ is a vector of manipulated variables and $\boldsymbol{V}(s)$ $[m\times1]$ is a vector of disturbance variables and it is considered $m \le n$.

Transfer function matrices of controlled plant $\boldsymbol{G}_S(s)$ and disturbance variables $\boldsymbol{G}_{SV}(s)$ are considered in form

$$\boldsymbol{G}_S(s) = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix}, \quad S_{ij} = \frac{Y_{S,i}(s)}{U_j(s)} \qquad (1)$$

where $i, j = <1, ..., n>$ and

$$\boldsymbol{G}_{SV}(s) = \begin{bmatrix} S_{V11} & S_{V12} & \cdots & S_{V1m} \\ S_{V21} & S_{V22} & \cdots & S_{V2m} \\ \vdots & \vdots & \cdots & \vdots \\ S_{Vn1} & S_{Vn2} & \cdots & S_{Vnm} \end{bmatrix}, \quad S_{V\,ij} = \frac{Y_{SV,i}(s)}{V_j(s)} \qquad (2)$$

where $i = <1, ..., n>$, $j = <1, ..., m>$, $m \le n$.

Transfer function matrices of controller $\boldsymbol{G}_R(s)$ and correction member $\boldsymbol{G}_{KC}(s)$ are considered in form

$$\boldsymbol{G}_R(s) = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{21} & R_{22} & \cdots & R_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ R_{n1} & R_{n2} & \cdots & R_{nn} \end{bmatrix}, \quad R_{ij} = \frac{U_{R,i}(s)}{E_j(s)} \qquad (3)$$

where $i, j = <1, ..., n>$ and

$$\boldsymbol{G}_{KC}(s) = \begin{bmatrix} KC_{11} & KC_{12} & \cdots & KC_{1m} \\ KC_{21} & KC_{22} & \cdots & KC_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ KC_{n1} & KC_{n2} & \cdots & KC_{nm} \end{bmatrix}, \quad KC_{ij} = \frac{U_{KC,i}(s)}{V_j(s)} \qquad (4)$$

where $i = <1, ..., n>$, $j = <1, ..., m>$, $m \le n$.

2.2 Decoupling of multi-variable control loop and invariance of multi-variable control loop

It is often required at synthesis of multi-variable control loop, beside stability and quality of control, that one control variable causes a change of only one corresponding controlled variable, i.e. elimination of effects of loop interactions. Such multi-variable control loop is called decoupled. Other requirement at synthesis of multi-variable control loop can also be elimination of influence of measurable disturbance variables on controlled variables. Such multi-variable control loop is called invariant. Control loop at which the influence of disturbance variables is eliminated only partially (e.g. only in steady state) are called approximately invariant. They are often called also invariant up to ε where ε is an error caused by incomplete elimination of influence of disturbances. Control loop at which the influence of disturbances on controlled variables is completely eliminated are called absolutely invariant. [1]

In order to ensure decoupling and invariance of multi-variable control loop a closed loop transfer function matrix $\boldsymbol{G}_{W/Y}(s)$ and transfer function matrix of disturbance variables $\boldsymbol{G}_{V/Y}(s)$ are used, therefore

$$\boldsymbol{G}_{W/Y}(s) = \left[\boldsymbol{I} + \boldsymbol{G}_S(s)\boldsymbol{G}_R(s)\right]^{-1}\boldsymbol{G}_S(s)\boldsymbol{G}_R(s) \qquad (5)$$

$$\boldsymbol{G}_{V/Y}(s) = \left[\boldsymbol{I} + \boldsymbol{G}_S(s)\boldsymbol{G}_R(s)\right]^{-1}\left[\boldsymbol{G}_{SV}(s) - \boldsymbol{G}_S(s)\boldsymbol{G}_{KC}(s)\right] \qquad (6)$$

For ensuring decoupling of control loop transfer function matrix $\boldsymbol{G}_{W/Y}(s)\cdot$must be a diagonal matrix. Because the sum and product of two diagonal matrices are diagonal matrices, and the inverse of diagonal matrix is also diagonal matrix, then the requirement can be ensured if transfer function matrix $\boldsymbol{G}_S(s)\cdot\boldsymbol{G}_R(s)$ is diagonal. This condition is realized if (7) is valid

$$\frac{R_{ij}}{R_{kj}} = \frac{s_{ji}}{s_{jk}} \qquad i, j, k = <1, \ldots, n>, \; s_{jk} \ne 0 \qquad (7)$$

where R_{ij}, R_{kj} are separate members of a transfer function matrix of controller $\boldsymbol{G}_R(s)$ and s_{ji}, s_{jk} are algebraic supplements of separate elements of a transfer function matrix of controlled plant $\boldsymbol{G}_S(s)$.

It is considered that diagonal, i.e. main controllers R_{ii} $(i = 1, 2, 3, \ldots, n)$, are already known from the first design of conception of control. These main controllers are designed via arbitrary SISO synthesis methods [1], [9]. Relation (7) is used to calculation of aside from diagonal members of a transfer function matrix of controller $\boldsymbol{G}_R(s)$ (binding members), which means that above mentioned relation can be rewritten into the following form

$$\frac{R_{ij}}{R_{jj}} = \frac{s_{ji}}{s_{jj}} \qquad i, j = <1, \ldots, n>, s_{jj} \neq 0 \tag{8}$$

The other strategy how to ensure decoupling of MIMO control loop can be found e.g. in [16]–[19].

Absolute invariance of control loop can be ensure if the transfer function matrix of disturbance variables $\boldsymbol{G}_{V/Y}(s)$ (6) is zero. This is possible if the following relation is valid

$$\boldsymbol{G}_{KC}(s) = \boldsymbol{G}_S^{-1}(s)\boldsymbol{G}_{SV}(s) \tag{9}$$

Correction members KC of transfer function matrix of correction members $\boldsymbol{G}_{KC}(s)$ can be determined from (10)

$$KC_{ij} = \frac{1}{\det \boldsymbol{G}_S} \sum_{k=1}^{n} s_{ki} \cdot S_{V,kj} \qquad \det \boldsymbol{G}_S \neq 0, i = <1, \ldots, n>, j = <1, \ldots, m>, m \leq n \tag{10}$$

where $\det \boldsymbol{G}_S$ is a determinant of transfer function matrix of controlled plant $\boldsymbol{G}_S(s)$, $S_{V,kj}$ are separate members of a transfer function matrix of disturbance variables $\boldsymbol{G}_{SV}(s)$ and s_{ki} are algebraic supplements of separate elements of a transfer function matrix of controlled plant $\boldsymbol{G}_S(s)$.

In case diagonal members of a transfer function matrix of disturbance variables $\boldsymbol{G}_{SV}(s)$ are considered as a dominant it is possible to simplify the above mentioned relation. In this case it is considered that internal couplings are omitted at MIMO control loop and thus n SISO branched control loops with measurement of a disturbance variable are gained. Connection of all SISO branched control loops is the same and they differ in separate transfer functions of controlled plants, controllers, correction members, disturbance variables, manipulated variables, setpoints and controlled variables (see Fig.2). [1]

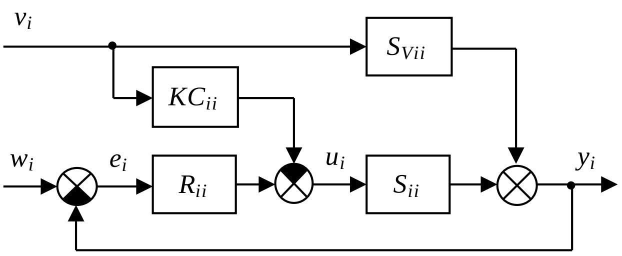

Fig. 2 - Single-variable branched control loop with measurement of disturbance variable

Transfer of correction members KC is then determined by using the following equation

$$KC_{ii} = \frac{S_{V,ii}}{S_{ii}} \quad i = <1, \ldots, n>, S_{ii} \neq 0 \qquad KC_{ij} = 0 \quad i \neq j,\ i = <1, \ldots, n>, j = <1, \ldots, m>, m \leq n \tag{11}$$

where $S_{V,ii}$ are separate members of transfer function matrix of disturbance variables $\boldsymbol{G}_{SV}(s)$ and S_{ii} are separate members of transfer function matrix of controlled plant $\boldsymbol{G}_S(s)$.

2.3 Control of multi-variable control loop

One of the possible methods of control of MIMO control loop is described in this part of the paper. The described method is demonstrated on simulation example of three-variable of control loop of steam turbine. The method of control is possible to divide into three basic parts, i.e. determination of parameters of main controllers then ensuring decoupling of control loop and finally ensuring invariance of control loop. [1]

Main controllers, which are diagonal elements of transfer function matrix of controller $\boldsymbol{G}_R(s)$, are designed by any synthesis method of SISO control loops. That means parameters of main controllers are determined for n SISO control loops (R_{11}, R_{22}, R_{33}, …, R_{nn}) by means of any synthesis method of SISO control loops. It is considered that original diagonal transfer functions S_{ii} ($i = 1, \ldots, n$) of transfer function matrix of controlled plant $\boldsymbol{G}_S(s)$ are modified to diagonal transfer functions $S_{ii,x}$ ($i = 1, \ldots, n$). In these modified transfer functions influences of aside-from-diagonal transfer functions of transfer function matrix of controlled plant $\boldsymbol{G}_S(s)$, i.e. S_{ij} ($i \neq j$, $i, j = 1, \ldots, n$) on original diagonal transfer functions, i.e. S_{ii} ($i = 1, \ldots, n$) are included. Modified transfer functions $S_{ii,x}$, i.e. $S_{11,x}$, $S_{22,x}$, $S_{33,x}$, etc. are determined from (12) by using (5) and (7).

$$S_{ii,x} = \sum_{j=1}^{n} S_{ij} \frac{s_{ij}}{s_{ii}} \qquad i = <1, \ldots, n>, s_{ii} \neq 0 \tag{12}$$

where s_{ii}, s_{ij} are algebraic supplements of separate elements of a transfer function matrix of controlled plant $\boldsymbol{G}_S(s)$ and S_{ij} are separate members of a transfer function matrix of controlled plant $\boldsymbol{G}_S(s)$.

Decoupling of control loop is ensured by means of binding members (8), which are aside from diagonal members of a transfer function matrix of controller $\boldsymbol{G}_R(s)$.

Invariance of control loop, which is elimination of influence of disturbance variables in the control loop, is ensured by means of correction members KC by using of equations (10) or (11). Relation (11) can be used when influences of aside from diagonal elements of a transfer function matrix of disturbance variables $\boldsymbol{G}_{SV}(s)$ are not dominant. In this case invariance of control loop is ensured by using n SISO branched control loops with measurement of disturbance variables.

3 Simulation Verification of Described Method of Control of Multi-variable Control Loop

3.1 Three-variable controlled plant of steam turbine

Typical example of MIMO controlled plant is a steam turbine [5], [6]. In this case it is considered the steam turbine with two controlled withdrawals which drives electric generator supplying determined part of electric network, which means the turbine operates without phasing into power network [1]. The scheme of three-variable controlled plant of steam turbine is shown in the Fig.3 [4].

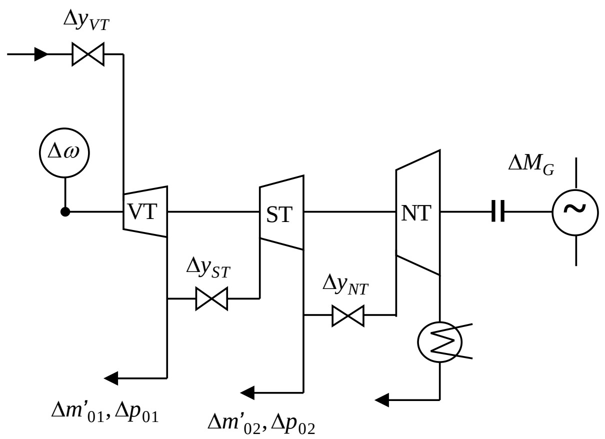

Fig. 3 - Three-variable control plant of steam turbine

Descriptions of separate parameters, which are in the scheme of three-variable control plant of steam turbine, are following: $\Delta\omega$ is a change of angular speed of turbo-generator, Δp_{01}, Δp_{02} are changes of steam pressures in corresponding withdrawals, Δy_{VT}, Δy_{ST}, Δy_{NT} are changes of opening position of control valves of high-pressure (VT), medium-pressure (ST) and low-pressure part of turbine (NT) , ΔM_G is a change of electric load of turbo-generator and $\Delta m'_{01}$, $\Delta m'_{02}$ are changes of mass flows of withdrawn steam. These parameters represent variables in three-variable control loop with measurement of disturbance variables.

Controlled variables (y_i) are parameters $\Delta\omega$, Δp_{01}, Δp_{02}, manipulated variables (u_i) are parameters Δy_{VT}, Δy_{ST}, Δy_{NT} and disturbance variables (v_i) are parameters ΔM_G, $\Delta m'_{01}$, $\Delta m'_{02}$.

3.2 Mathematical model of three-variable controlled plant of steam turbine

Mathematical model of the controlled plant of steam turbine is given by three differential equations (13) - (15). These differential equations were gained already after deriving and using linearization from project OTROKOVICE elaborated by the firm ALSTOM Power [4].

The first differential equation represents moment balance which is in the form

$$518.4\Delta\dot{\omega} = -63.3\Delta\omega + 656.9\Delta p_{01} + 4611.7\Delta p_{02} + 1007.3\Delta y_{VT} + 200.6\Delta y_{ST} + 121.5\Delta y_{NT} - \Delta M_G \quad (13)$$

second and third differential equations represent flow through flow spaces which are in forms

$$1.865\Delta\dot{p}_{01} = -1.610\Delta p_{01} + 0.167\Delta p_{02} + 1.523\Delta y_{VT} - 0.361\Delta y_{ST} - \Delta m'_{01} \quad (14)$$

$$13.45\Delta\dot{p}_{02} = 1.563\Delta p_{01} - 10.517\Delta p_{02} + 0.361\Delta y_{ST} - 0.222\Delta y_{NT} - \Delta m'_{02}. \quad (15)$$

It is possible to rewrite the above differential equations (13) - (15) into better form (18) - (20) by introducing relative values, i.e. with regard to starting stable state-operational, the so called calculated point, at which relation of values can be generally written in the form

$$\varphi_X = \frac{\Delta X}{(X)_0} \rightarrow \Delta X = \varphi_X \cdot (X)_0 \quad (16)$$

where separate operational parameters of controlled plant of steam turbine in the calculated point are following

$$(\omega)_0 = 628.3\,[\text{rad/s}],\ (p_{01})_0 = 14\,[\text{bar}],\ (p_{02})_0 = 1.55\,[\text{bar}],\ (y_{VT})_0 = 19.15\,[\text{mm}],\ (y_{ST})_0 = 59.9\,[\text{mm}],\ (y_{NT})_0 = 69.8\,[\text{mm}],\ (M_G)_0 = 39789\,[\text{Nm}],\ (m'_{01})_0 = 6.94\,[\text{kg/s}],\ (m'_{02})_0 = 6.94\,[\text{kg/s}] \quad (17)$$

hence

$$325710.7\dot{\varphi}_{\omega} = -39771.4\varphi_{\omega} + 9196.6\varphi_{p_{01}} + 7148.1\varphi_{p_{02}} + 19289.8\varphi_{y_{VT}} + 12015.9\varphi_{y_{ST}} + 8480.7\varphi_{y_{NT}} - 39789\,\varphi_{M_G} \quad (18)$$

$$26.110\dot{\varphi}_{p_{01}} = -22.540\varphi_{p_{01}} + 0.259\varphi_{p_{02}} + 29.165\varphi_{y_{VT}} - 21.624\varphi_{y_{ST}} - 6.94\varphi_{m'_{01}} \quad (19)$$

$$20.848\dot{\varphi}_{p_{02}} = 21.882\varphi_{p_{01}} - 16.301\varphi_{p_{02}} + 21.624\varphi_{y_{ST}} - 15.496\varphi_{y_{NT}} - 6.94\varphi_{m'_{02}}. \quad (20)$$

In the next step it is carried out the Laplace transform of the modified differential equations

(18) - (20). They are gained three algebraic equations, out of which after arrangement it is possible to put together transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$ (23) and transfer function matrix of disturbance variables $\boldsymbol{G}_{SV}(s)$ (24). The Laplace transform of a vector of output variables, i.e. vector of controlled variables is generally given by (21).

$$\boldsymbol{Y}(s) = \boldsymbol{G}_S(s)\boldsymbol{U}(s) + \boldsymbol{G}_{SV}(s)\boldsymbol{V}(s) \tag{21}$$

where $\boldsymbol{Y}(s)$ is a vector of controlled variables, $\boldsymbol{U}(s)$ is vector of manipulated variables and $\boldsymbol{V}(s)$ is a vector of disturbance variables.

Further it is considered $\boldsymbol{Y}(s) = [\Phi_\omega, \Phi_{p_{01}}, \Phi_{p_{02}}]^{\mathrm{T}}$, $\boldsymbol{U}(s) = [\Phi_{y_{VT}}, \Phi_{y_{ST}}, \Phi_{y_{NT}}]^{\mathrm{T}}$ and $\boldsymbol{V}(s) = [\Phi_{M_G}, \Phi_{m'_{01}}, \Phi_{m'_{02}}]^{\mathrm{T}}$. After substitution it is possible to re-write (21) in the following form (22).

$$\begin{bmatrix} \Phi_\omega \\ \Phi_{p_{01}} \\ \Phi_{p_{02}} \end{bmatrix} = \boldsymbol{G}_S(s) \begin{bmatrix} \Phi_{y_{VT}} \\ \Phi_{y_{ST}} \\ \Phi_{y_{NT}} \end{bmatrix} + \boldsymbol{G}_{SV}(s) \begin{bmatrix} \Phi_{M_G} \\ \Phi_{m'_{01}} \\ \Phi_{m'_{02}} \end{bmatrix} \tag{22}$$

where

$$\boldsymbol{G}_S(s) = \begin{bmatrix} \frac{0.73s^2+1.59s+1.11}{12.3s^3+21.8s^2+10.7s+1} & \frac{0.46s^2+0.74s+0.09}{12.3s^3+21.8s^2+10.7s+1} & \frac{0.32s^2+0.32s+0.04}{12.3s^3+21.8s^2+10.7s+1} \\ \frac{1.68s+1.31}{1.51s^2+2.48s+1} & \frac{-1.25s-0.96}{1.51s^2+2.48s+1} & \frac{-0.01}{1.51s^2+2.48s+1} \\ \frac{1.76}{1.51s^2+2.48s+1} & \frac{1.56s+0.040}{1.51s^2+2.48s+1} & \frac{-1.12s-0.97}{1.51s^2+2.48s+1} \end{bmatrix} \tag{23}$$

$$\boldsymbol{G}_{SV}(s) = \begin{bmatrix} \frac{-1.51s^2-2.48s-1}{12.3s^3+21.8s^2+10.7s+1} & \frac{-0.09s-0.15}{12.3s^3+21.8s^2+10.7s+1} & \frac{-0.09s-0.08}{12.3s^3+21.8s^2+10.7s+1} \\ 0 & \frac{-0.40s-0.31}{1.51s^2+2.48s+1} & \frac{-0.005}{1.51s^2+2.48s+1} \\ 0 & \frac{-0.420}{1.51s^2+2.48s+1} & \frac{-0.501s-0.432}{1.51s^2+2.48s+1} \end{bmatrix} \tag{24}$$

Step response and impulse response of transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$ (23) and transfer function matrix of disturbance variables $\boldsymbol{G}_{SV}(s)$ (24) are shown in the following figures (see Fig. 4 - Fig. 7).

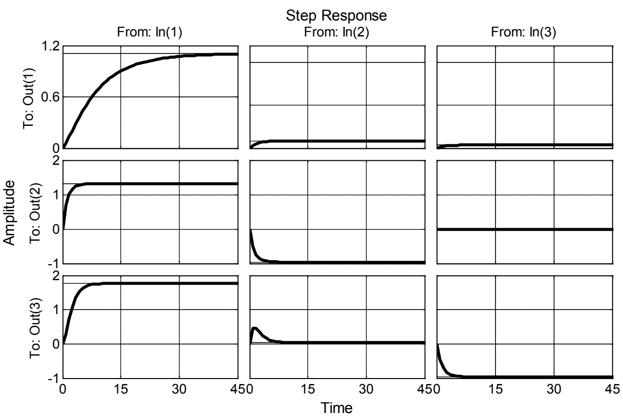

Fig. 4 - Step response of transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$ (23)

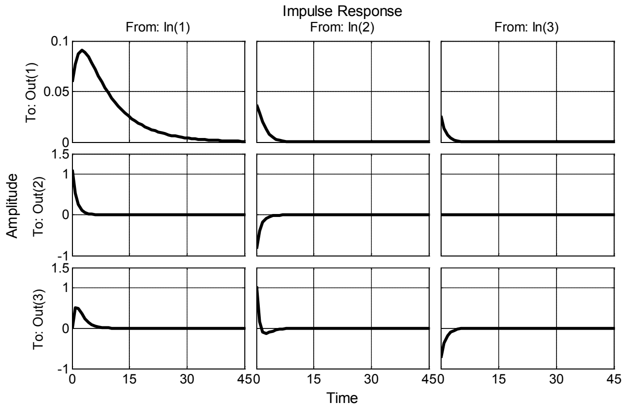

Fig. 5 - Impulse response of transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$ (23)

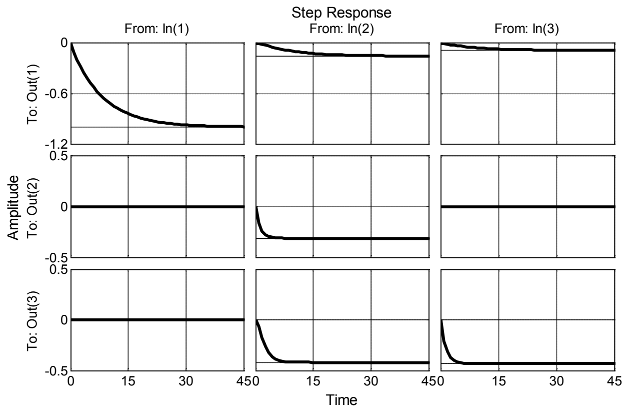

Fig. 6 - Step response of transfer function matrix of the disturbance variables $\boldsymbol{G}_{SV}(s)$ (24)

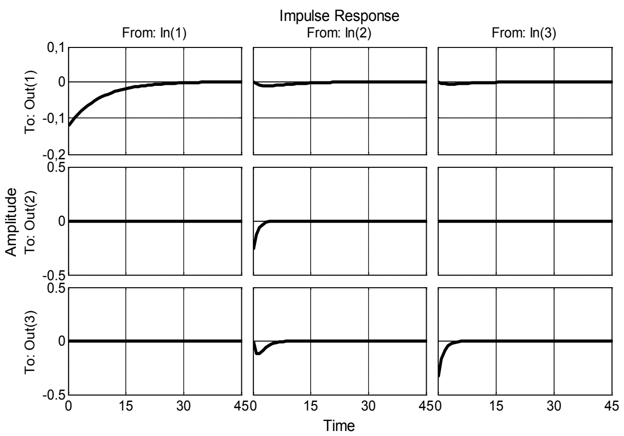

Fig. 7 - Impulse response of transfer function matrix the disturbance variables $\boldsymbol{G}_{SV}(s)$ (24)

3.3 Control of three-variable control loop of a steam turbine

The procedure of control described in the paragraph "Control of multi-variable control loop" is used at control of the three-variable control loop of a steam turbine. First of all transfer functions of main controllers R_{11}, R_{22}, R_{33} are determined by means of arbitrary SISO synthesis method for modified diagonal transfer functions $S_{11,x}$, $S_{22,x}$ a $S_{33,x}$ (12). After that binding members are determined by using of (8). This relation serves to ensuring decoupling of control loop. Finally parameters of correction members, which ensured invariance of control loop, are calculated by using of (10) (absolute invariance) or (11) (approximate invariance).

To determination of modified transfer functions $S_{ii,x}$ (i = 1, 2, 3) was used (12), then

$$\begin{aligned} S_{11,x}(s) &= S_{11}\frac{s_{11}}{s_{11}} + S_{12}\frac{s_{12}}{s_{11}} + S_{13}\frac{s_{13}}{s_{11}} = \\ &= S_{11} + S_{12}\frac{S_{23}S_{31} - S_{21}S_{33}}{S_{22}S_{33} - S_{23}S_{32}} + S_{13}\frac{S_{21}S_{32} - S_{22}S_{31}}{S_{22}S_{33} - S_{23}S_{32}} \\ S_{22,x}(s) &= S_{21}\frac{s_{21}}{s_{22}} + S_{22}\frac{s_{22}}{s_{22}} + S_{23}\frac{s_{23}}{s_{22}} = \\ &= S_{21}\frac{S_{13}S_{32} - S_{12}S_{33}}{S_{11}S_{33} - S_{13}S_{31}} + S_{22} + S_{23}\frac{S_{12}S_{31} - S_{11}S_{32}}{S_{11}S_{33} - S_{13}S_{31}} \\ S_{33,x}(s) &= S_{31}\frac{s_{31}}{s_{33}} + S_{32}\frac{s_{32}}{s_{33}} + S_{33}\frac{s_{33}}{s_{33}} = \\ &= S_{31}\frac{S_{12}S_{23} - S_{13}S_{22}}{S_{11}S_{22} - S_{12}S_{21}} + S_{32}\frac{S_{13}S_{21} - S_{11}S_{33}}{S_{11}S_{22} - S_{12}S_{21}} + S_{33} \end{aligned} \qquad (25)$$

hence

$$S_{11,x} = \frac{1.294}{8.19s+1},\ S_{22,x} = \frac{-1.057}{0.479s+1},\ S_{33,x} = \frac{-1.02}{0.946s+1}. \qquad (26)$$

At design of parameters of main controllers R_{11}, R_{22}, and R_{33}, which are diagonal elements of transfer function matrix of controller $\boldsymbol{G}_R(s)$, the following SISO synthesis methods were used

a) Whiteley method [1], [12]
b) method of optimal module [1], [8]
c) method of desired model [8]
d) pole placement method [9], [11].

It is used a polynomial approach at design of parameters of main controllers by means of pole placement method. Further it is considered that roots of the closed control loop (poles), which influence quality and stability of the closed control loop, are selected like multiple roots.

It is possible to use also other methods of parameters design of main (diagonal) controllers of transfer function matrix of controller $\boldsymbol{G}_R(s)$, e.g. Ziegler-Nichols methods, Naslin method, the symmetrical optimum method, SIMC method, etc. [1], [7]–[12].

To calculation of binding members R_{ij} ($i \neq j$), which ensuring decoupling of control loop, was used (8). It means these binding members were gained from the following relations

$$\begin{aligned} R_{12}(s) &= \frac{s_{21}}{s_{22}}R_{22} = \frac{S_{13}S_{32} - S_{12}S_{33}}{S_{11}S_{33} - S_{13}S_{31}}R_{22} \\ R_{13}(s) &= \frac{s_{31}}{s_{33}}R_{33} = \frac{S_{12}S_{23} - S_{13}S_{22}}{S_{11}S_{22} - S_{12}S_{21}}R_{33} \\ R_{21}(s) &= \frac{s_{12}}{s_{11}}R_{11} = \frac{S_{23}S_{31} - S_{21}S_{33}}{S_{22}S_{33} - S_{23}S_{32}}R_{11} \\ R_{23}(s) &= \frac{s_{32}}{s_{33}}R_{33} = \frac{S_{13}S_{21} - S_{11}S_{33}}{S_{11}S_{22} - S_{12}S_{21}}R_{33} \\ R_{31}(s) &= \frac{s_{13}}{s_{11}}R_{11} = \frac{S_{21}S_{32} - S_{22}S_{31}}{S_{22}S_{33} - S_{23}S_{32}}R_{11} \\ R_{32}(s) &= \frac{s_{23}}{s_{22}}R_{22} = \frac{S_{12}S_{31} - S_{11}S_{32}}{S_{11}S_{33} - S_{13}S_{31}}R_{22} \end{aligned} \qquad (27)$$

To determine of correction members KC, which ensuring invariance of control loop, was used (10) and (11). Relation (10) ensures absolute invariance of control loop, e.g. the first four correction members are following

$$\begin{aligned} KC_{11}(s) &= \frac{1}{\det \boldsymbol{G}_S}\left(s_{11}S_{V,11} + s_{21}S_{V,21} + s_{31}S_{V,31}\right) \\ KC_{12}(s) &= \frac{1}{\det \boldsymbol{G}_S}\left(s_{11}S_{V,12} + s_{21}S_{V,22} + s_{31}S_{V,32}\right) \\ KC_{13}(s) &= \frac{1}{\det \boldsymbol{G}_S}\left(s_{11}S_{V,13} + s_{21}S_{V,23} + s_{31}S_{V,33}\right) \\ KC_{21}(s) &= \frac{1}{\det \boldsymbol{G}_S}\left(s_{12}S_{V,11} + s_{22}S_{V,21} + s_{32}S_{V,31}\right) \\ &\vdots \end{aligned} \qquad (28)$$

where $\det \boldsymbol{G}_S$ is a determinant of transfer function matrix of controlled plant $\boldsymbol{G}_S(s)$ and s_{ki} (k, i = 1, 2, 3) are algebraic supplements of separate elements of a transfer function matrix of controlled plant $\boldsymbol{G}_S(s)$ (see (25) or (27)).

Relation (11) ensures that control loop is approximately invariant, hence

$$\begin{aligned} &KC_{11} = \frac{S_{V11}}{S_{11}},\ KC_{22} = \frac{S_{V22}}{S_{22}},\ KC_{33} = \frac{S_{V33}}{S_{33}} \\ &KC_{ij} = 0 \qquad i \neq j,\ \ i, j = <1, 2, 3>. \end{aligned} \qquad (29)$$

Transfer function matrix of controllers $\boldsymbol{G}_R(s)$ with utilization of four chosen SISO synthesis methods for design of parameters of main controllers is following

a) Whiteley method

$$G_R(s)=\begin{bmatrix} \frac{0.0482}{s} & \frac{1.246\,s+0.160}{s(s+2.089)} & \frac{0.126\,s+0.0165}{s(s+1.057)} \\ \frac{0.0649}{s} & \frac{-1.008}{s} & \frac{0.170\,s+0.0291}{s(s+1.057)} \\ \frac{0.0906}{s} & \frac{-1.407s+0.2059}{s(s+2.089)} & \frac{-0.529}{s} \end{bmatrix} \quad (30)$$

b) method of optimal module

$$G_R(s)=\begin{bmatrix} \frac{0.0472}{s} & \frac{1.222\,s+0.156}{s(s+2.089)} & \frac{0.124\,s+0.0162}{s(s+1.057)} \\ \frac{0.0636}{s} & \frac{-0.988}{s} & \frac{0.167\,s+0.0285}{s(s+1.057)} \\ \frac{0.0888}{s} & \frac{-1.379\,s+0.202}{s(s+2.089)} & \frac{-0.518}{s} \end{bmatrix} \quad (31)$$

c) method of desired model

$$G_R(s)=\begin{bmatrix} \frac{0.633\,s+0.0773}{s} & \frac{0.112\,s+0.0143}{s(s+2.089)} & \frac{0.0317\,s+0.00413}{s(s+1.057)} \\ \frac{0.854\,s+0.104}{s} & \frac{-0.0905\,s-0.189}{s} & \frac{0.0427\,s+0.00728}{s(s+1.057)} \\ \frac{1.191\,s+0.146}{s} & \frac{-0.126\,s+0.0185}{s(s+2.089)} & \frac{-0.133\,s-0.140}{s} \end{bmatrix} \quad (32)$$

d) pole placement method

$$G_R(s)=\begin{bmatrix} \frac{1.632\,s+0.226}{s} & \frac{0.71\,s^2+0.378\,s+0.0367}{s(s+2.089)} & \frac{0.30\,s^2+0.179\,s+0.0182}{s(s+1.057)} \\ \frac{2.202\,s+0.305}{s} & \frac{-0.575\,s-0.232}{s} & \frac{0.40\,s^2+0.257\,s+0.0321}{s(s+1.057)} \\ \frac{3.072\,s+0.425}{s} & \frac{-0.80\,s^2-0.206\,s+0.0473}{s(s+2.089)} & \frac{-1.246\,s-0.585}{s} \end{bmatrix} \quad (33)$$

where aside from diagonal elements of transfer function matrix of controllers $\boldsymbol{G}_R(s)$ was calculated from (27).

Transfer function matrix of correction members $\boldsymbol{G}_{KC}(s)$ is given by the relation (28) or (29). Corresponding relation is always used for all simulation experiments. Relation (28) ensures absolute invariance of control loop, hence

$$\boldsymbol{G}_{KC}(s)=\begin{bmatrix} -0.773 & -0.149 & -0.074 \\ -1.043 & 0.120 & 0.100 \\ -1.455 & 0.168 & 0.309 \end{bmatrix} \quad (34)$$

Relation (29) ensures that control loop is approximately invariant, hence

$$\boldsymbol{G}_{KC}(s)=\begin{bmatrix} KC_{11} & 0 & 0 \\ 0 & KC_{22} & 0 \\ 0 & 0 & KC_{33} \end{bmatrix}$$

$$KC_{11}=\frac{-2.063s^2-3.394s-1.371}{s^2+2.178s+1.516}, \quad (35)$$

$$KC_{22}=\frac{0.321s+0.251}{s+0.769}, \qquad KC_{33}=0.448$$

The scheme of three-variable control loop is generally considered according to the following figure (see Fig. 8).

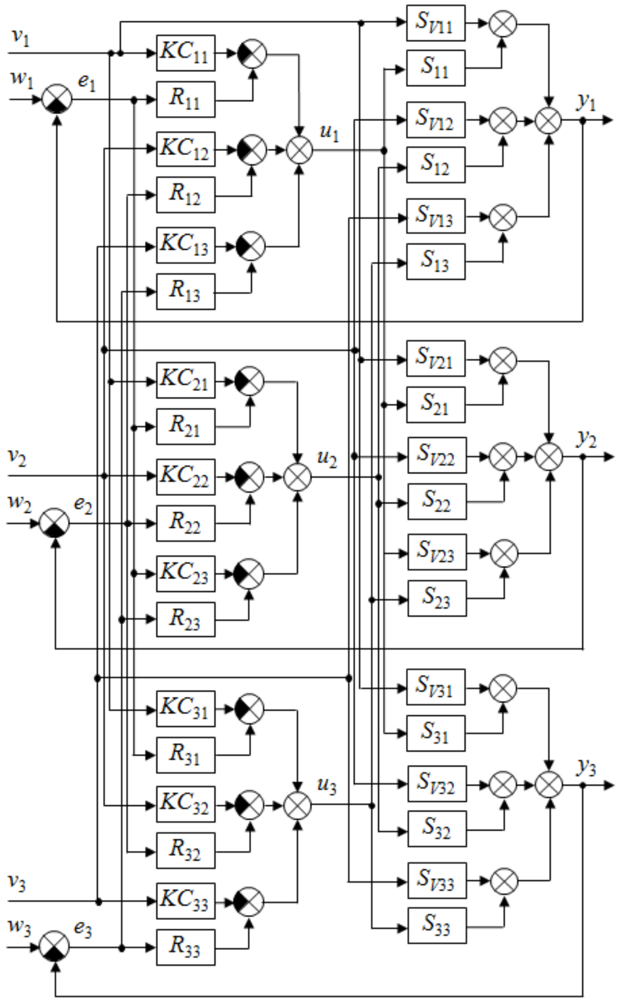

Fig. 8 - Three-variable branched control loop with measurement of disturbance variables

3.4 Simulation results

Mathematical software MATLAB/SIMULINK [12] is used for simulating verification of proposed control method of MIMO control loop. Simulation scheme shown in the Fig. 9 is utilized for these purposes.

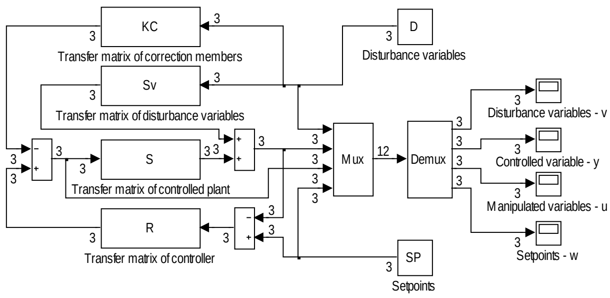

Fig. 9 - Simulation scheme of three-variable control loop in the program MATLAB/SIMULINK

Simulation courses of three-variable control loop of a steam turbine, with utilization of chosen SISO synthesis methods at design of parameters of main controllers, are presented in the following figures (see Fig. 10 - Fig. 17). Fig. 10 - Fig. 13 show simulation courses of three-variable control loop where absolute invariance (28) is ensured. Fig. 14 - Fig. 17 show simulation courses of three-variable control loop where approximate invariance (29) is ensured.

The following parameters were chosen and used at all simulation experiments (see Fig. 10 - Fig. 17)

- setpoints time vector (t_{w1}, t_{w2}, t_{w3}): [20, 220, 400]
- setpoints vector (w_1, w_2, w_3): [0.8, 0.8, 0.8]
- disturbances time vector (t_{v1}, t_{v2}, t_{v3}): [140, 320, 500]
- disturbances vector (v_1, v_2, v_3): [0.5, 0.5, 0.5]
- time step (k): 0.05
- total simulation time (t_S): 600

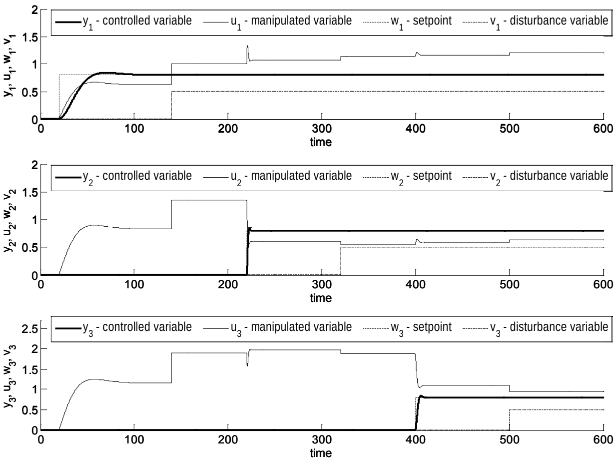

Fig. 10 - Simulation courses of control loop with utilization of Whiteley method for design of parameters of main controllers by ensuring absolute invariance of control loop via (34)

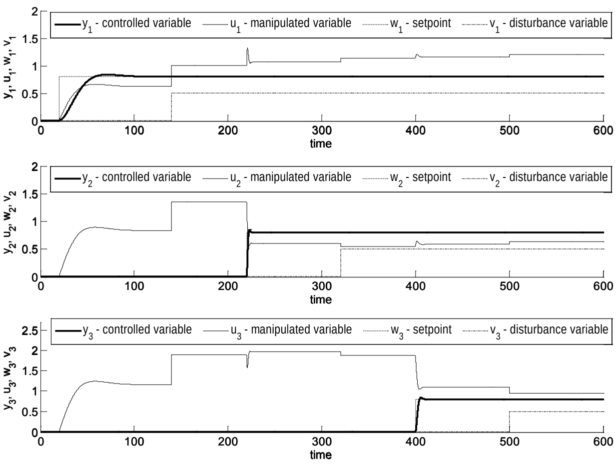

Fig. 11 - Simulation courses of control loop with utilization of method of optimal module for design of parameters of main controllers by ensuring absolute invariance of control loop via (34)

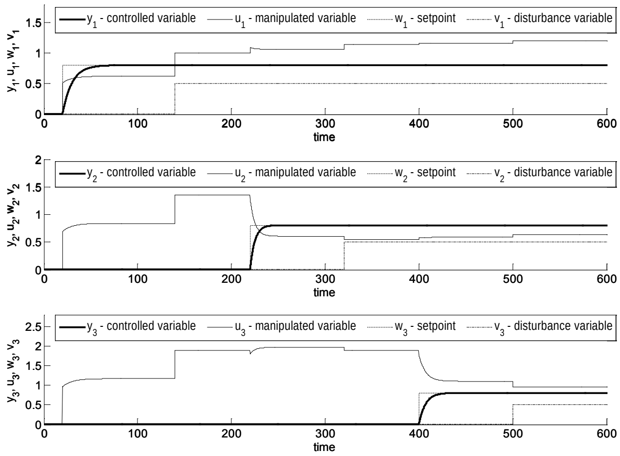

Fig. 12 - Simulation courses of control loop with utilization of method of desired model for design of parameters of main controllers by ensuring absolute invariance of control loop via (34)

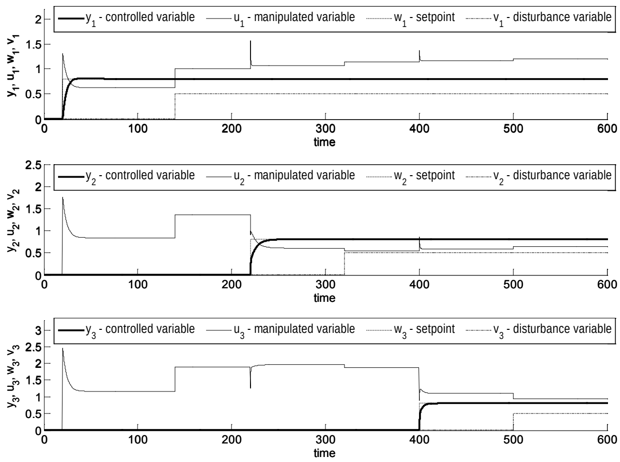

Fig. 13 - Simulation courses of control loop with utilization of pole placement method for design of parameters of main controllers by ensuring absolute invariance of control loop via (34)

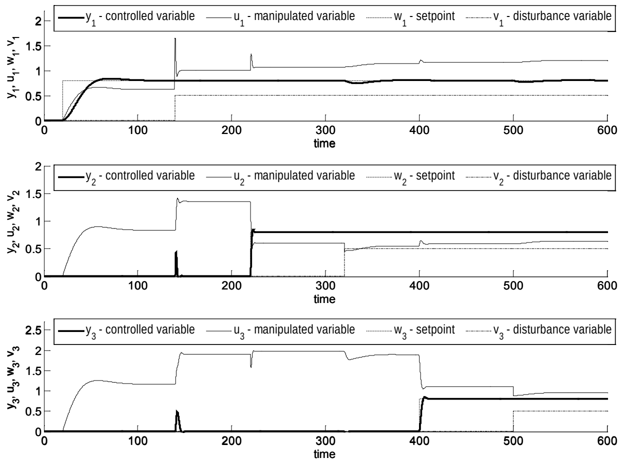

Fig. 14 - Simulation courses of control loop with utilization of Whiteley method for design of parameters of main controllers by ensuring approximate invariance of control loop via (35)

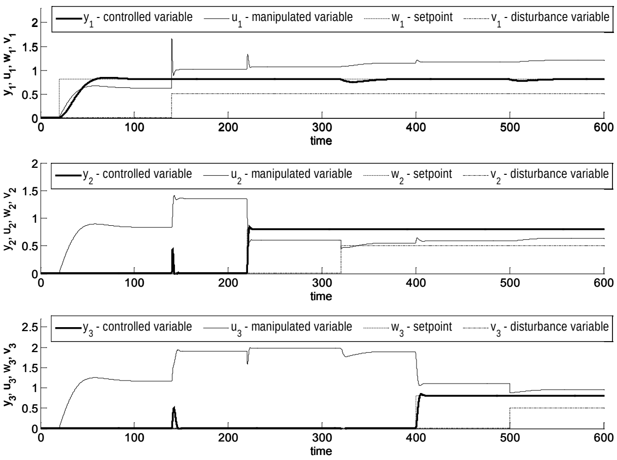

Fig. 15 - Simulation courses of control loop with utilization of method of optimal module for design of parameters of main controllers by ensuring approximate invariance of control loop via (35)

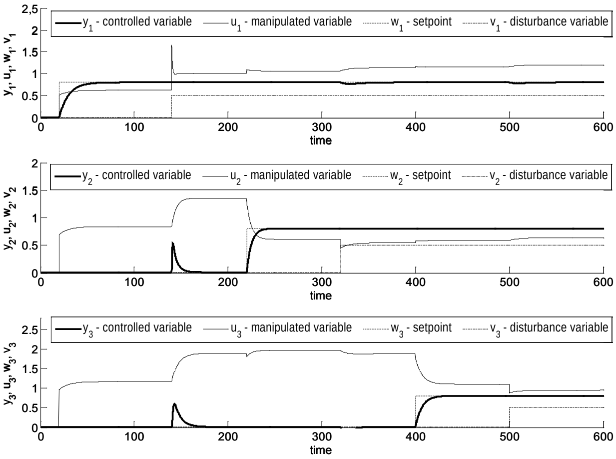

Fig. 16 - Simulation courses of control loop with utilization of method of desired model for design of parameters of main controllers by ensuring approximate invariance of control loop via (35)

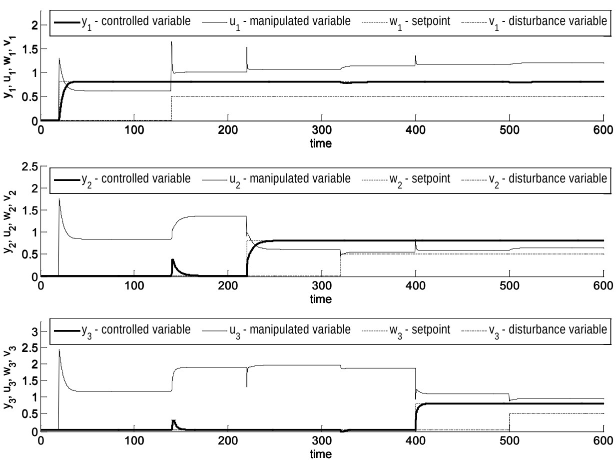

Fig. 17 - Simulation courses of control loop with utilization of pole placement method for design of parameters of main controllers by ensuring approximate invariance of control loop via (35)

Variables in the figures (see Fig. 10 - Fig. 17) correspond to variables described in the three-variable control loop of steam turbine (see Fig. 3), i.e.

- controlled variable: $y_1 \to \varphi_\omega$, $y_2 \to \varphi_{p_{01}}$, $y_3 \to \varphi_{p_{02}}$
- manipulated variable: $u_1 \to \varphi_{y_{VT}}, u_2 \to \varphi_{y_{ST}}, u_3 \to \varphi_{y_{NT}}$
- setpoints: $w_1 \to \varphi_\omega$, $w_2 \to \varphi_{p_{01}}$, $w_3 \to \varphi_{p_{02}}$
- disturbance variable: $v_1 \to \varphi_{M_G}, v_2 \to \varphi_{m'_{01}}, v_3 \to \varphi_{m'_{02}}$

3.5 Evaluation of simulation experiments obtained by using of described method of control of multi-variable control loop

To comparison simulation experiments the ISE criterion (36) and ITAE criterion (37) were used (see Table 1 and Table 2), i.e.

$$J_K = \text{ISE} = \int_0^\infty e^2(t)dt = \int_0^\infty \left[w(t) - y(t)\right]^2 dt \approx \int_0^{t_S} e^2(t)dt \quad (36)$$

$$J_K = \text{ITAE} = \int_0^\infty t \cdot \left|e(t)\right| dt = \int_0^\infty t \cdot \left|\left(w(t) - y(t)\right)\right| dt \approx \int_0^{t_S} t \cdot \left|e(t)\right| dt \quad (37)$$

where t_r is time of control, t_s is time of simulation, $w(t)$ is setpoint, $y(t)$ is controlled variable, $e(t)$ is control error (see Fig. 18).

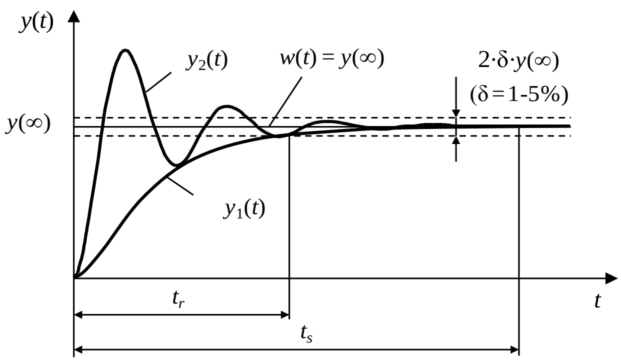

Fig. 18 - Possible courses of control loop

Table 1 - Quality of control for simulation example of three-variable control loop by ensuring invariance of control loop via (34)

Synthesis method *)	J_{K1}		J_{K2}		J_{K3}	
	ISE	ITAE	ISE	ITAE	ISE	ITAE
1	7.773	505.8	0.469	195.1	0.912	693.4
2	7.878	512.4	0.475	197.2	0.924	701.2
3	3.216	240.8	1.616	904.5	2.256	2287
4	1.210	69.43	0.928	742.6	0.386	548.7

Table 2 - Quality of control for simulation example of three-variable control loop by ensuring invariance of control loop via (35)

Synthesis method *)	J_{K1}		J_{K2}		J_{K3}	
	ISE	ITAE	ISE	ITAE	ISE	ITAE
1	7.853	1471	0.669	296.0	1.499	1059
2	7.959	1493	0.679	299.5	1.524	1073
3	3.243	786.8	2.488	1332	4.328	3607
4	1.215	252.6	1.396	1093	0.598	845.1

*) Numbers in the previous tables (Table 1 and Table 2) in the column "Synthesis method" represent the used SISO synthesis method of at design of parameters of main controllers, i.e.: 1 - Whiteley method, 2 - method of optimal module, 3 - method of desired model, 4 - pole placement method.

Optimal adjustment of control loop is considered here from the point of view of minimal size of ISE criterion or ITAE criterion (Table 1 and Table 2). However quite different point of view can be rally considered for optimal adjustment. Namely requirements for the smallest overshooting and for the shortest time of control are generally valid for optimal adjustment. However these requirements are antagonistic and therefore the optimal adjustment of controller is always a compromise between them.

Degree of internal coupling of MIMO controlled plant is often evaluated via RGA (Relative Gain Array) [18], [20], [21]. RGA values are depended on frequency. These values are usually determined for frequency equal to zero, i.e. for steady state. From the point of view of control, it is ideal state when values of diagonal elements of RGA matrix are approaching to the value of one and aside-from-diagonal elements of RGA matrix approaching to the value of zero. The RGA matrix ($\boldsymbol{\Lambda}$) can be calculated from the following equation

$$\boldsymbol{\Lambda} = \boldsymbol{G}(\mathrm{j}\omega) \otimes (\boldsymbol{G}^{-1}(\mathrm{j}\omega))^T = \boldsymbol{G}(0) \otimes (\boldsymbol{G}^{-1}(0))^T \quad (38)$$

where $\boldsymbol{G}(s)$ is a transfer function matrix of examined object ($s = \mathrm{j}\omega$), e.g. MIMO controlled plant, MIMO closed loop, $\otimes$ operator implies an element by element multiplication (Schur product).

It is possible to use RGA values to compare properties of original MIMO controlled plant and MIMO control loop from the point of view degree internal coupling.

RGA matrix of three-variable controlled plant is following

$$\boldsymbol{\Lambda}(\boldsymbol{G}_S(0)) = \begin{bmatrix} 0.8548 & 0.0912 & 0.0540 \\ 0.0942 & 0.9068 & -0.0010 \\ 0.0510 & 0.0020 & 0.9470 \end{bmatrix} \quad (39)$$

and RGA matrix of closed loop transfer function matrix $\boldsymbol{G}_{W/Y}(s)$ of three-variable control loop, for all MIMO controller (30) - (33), is in the form

$$\boldsymbol{\Lambda}(\boldsymbol{G}_{W/Y}(0)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (40)$$

It is obvious from the simulations of control loop shown above (see Fig. 10 - Fig. 17) that the condition of decoupling of control loop was fulfilled. Fulfilment of this condition was ensured via binding members, which are aside-from-diagonal elements of transfer function matrix of controller $\boldsymbol{G}_R(s)$ (30) - (33). Binding members were determined from so called main controllers, which are main diagonal elements of the transfer function matrix of controller. The determination of main controllers is carried out by any SISO synthesis method for modified diagonal elements of the transfer function matrix of controlled plant.

From the simulation of control loop is also obvious that the control loop is absolute invariant (see Fig. 10 - Fig. 13 and (34)), let us say, approximate invariant (see Fig. 14 - Fig. 17 and (35)). In this second case influence of disturbance variables is eliminated only at steady state. Fulfilment of this condition was ensured via separate elements of transfer function matrix of correction members $\boldsymbol{G}_{KC}(s)$ (34) - (35), i.e. via correction members *KC*. Correction members were determined from transfer function matrix of the controlled plant and disturbance variables.

4 Conclusion

In this paper was to described and shown the one of possible approaches to control of MIMO control loops. Advantage of described and used control method is its simplicity. This control method enables to use any known SISO synthesis method to design of main controllers. The control method combines classical way of ensuring decoupling of control loop via binding members, which are aside-from-diagonal elements of transfer function matrix of controller $\boldsymbol{G}_R(s)$, and the use of the correction members for ensuring absolute invariance or approximate invariance of MIMO control loop. Simulation verification of proposed control method was presented on three-variable control loop of steam turbine.

Designed parameters of matrix controllers and correction members have good results of the control and fulfilled basic control requirements such as the stability, the reference signal tracking and disturbance attenuation.

The described and used control method is valid under the following condition, i.e. this method can be used only for MIMO controlled system with same number of input and output signals. MIMO controlled plant containing transport delay, non-minimal phase or having high order dynamics can be also cause of certain limitations of the control method.

The future work will be focused on the reduction of limitations of proposed control method, verification of alternative approach to ensuring decoupling of control loop [19] and also simulation verification of proposed, let us say, modified version of control method for other MIMO controlled plants, e.g. model of balance platform system [22], model of heating system [23].

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