Self – Tuning Control of Continuous – Time Systems

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Abstract: - One approach to a recursive identification of continuous – time systems was implemented in self – tuning control of a system of interconnected tanks. Since derivatives of input and output variables of continuous – time systems can not be directly measured, differential filters and filtered variables are established to substitute primary variables. The filtered variables are then used in a recursive identification procedure where the classical recursive least squares method is used to identify the system. Results of real – time experiments are compared to results obtained with an analogical discrete controller. The method was also verified by simulation on a multivariable system in order to verify the algorithm on a system with more complex and complicated structure, where one output is influenced by more inputs.

Key-Words: - self – tuning control, adaptive control, recursive estimation, nonlinear systems, polynomial methods, real – time control

1 Introduction

A significant part of technological processes in industrial practice has a stochastic character. Traditional controllers with fixed parameters are often unsuitable for such processes, because parameters of the process vary in time and control can not be optimal. This leads to losses of materials and energy, wear of machinery and so on. Parameter changes are caused by changes in a manufacturing process and controllers with fixed parameters can not deal with this. A possible method for control of systems with variable parameters is adaptive control using self – tuning controllers [1]. Other two basic classes of adaptive controllers are controllers based on the heuristic approach and model reference adaptive systems. These two approaches are limited by the fact that they are only suited to control of deterministic systems. That is why self – tuning controllers have better prospects for their wider use in industrial practice. Another wide range of processes belong to a class of nonlinear systems. A nonlinear dynamic behaviour often causes conventional control methods to be unsatisfactory. This problem can be also solved by using self – tuning control. The nonlinear dynamics are described by a linear model in the neighbourhood of a steady state [2].

Self – tuning controllers are based on recursive identification of an unknown process on the basis of input and output variables and consequent synthesis of a controller. Internal structure of a self – tuning controller is divided into two parts – an identification part and a control part. The identification part is created by an adaptive predictor, which is an internal model of a controlled system. There are recursively actualised parameters of a model of a process, which are used for computation of the system output prediction. The control part contains a block for computation of controller's parameters, which are used for computation of manipulated variables in each sampling period. This approach had beginning in [3], where a special – purpose computer for an identification of parameters of a linear model with subsequent computation of a control law based on minimization of the quadratic criterion was proposed. The part of the recursive identification is as important as the part of a synthesis of a controller. For purposes of the adaptive control only those methods of identification are interesting, which are realizable in real – time.

In this paper, application of one approach to recursive identification of continuous – time system parameters is presented. Since input and output derivatives of a system can not be directly measured, the differential filters and filtered variables are established to substitute primary variables. This approach is described in detail in [4], [5], [6]. The filtered variables are then used in the recursive identification procedure, where the classical recursive least squares method is used to identify the parameters.

Whilst methods of design and applications of discrete adaptive controllers are frequently presented in literature, for example [7], [8], [9], self – tuning control of continuous – time systems is not widespread and there still is not much experience with continuous – time system parameter estimation. Some theoretical aspects of continuous – time systems identification are given for example in [10]. Limitations during control of systems with fast dynamics when discrete models with small sampling periods have poor numerical attributes are mostly solved using delta models [11]. An alternative solution is using of the method applied in this paper when a continuous time model is recursively estimated and design of a controller is performed in the continuous – time domain. This approach enables fast sampling. The value of the sampling period is then dependant only on capabilities of used hardware and software. The used software must enable realization of filters by differential equations.

A continuous - time self – tuning controller with 2dof (two degree of freedom) configuration, when a controller contains both feedback and feedforward parts [12], was implemented for real – time control of a model of interconnected tanks. The model is a nonlinear system with variable parameters. The results of control were compared to results obtained with an analogical self – tuning discrete controller. Design of the controllers was based on polynomial methods [13].

Verification of the method was also carried out on a two input/two output system. This case was chosen in order to verify the algorithm on a system with more complex and complicated structure, where one output is influenced by more inputs. Moreover, most of technological processes require that several variables relating to one system are controlled simultaneously. Each input may influence all system outputs.

2 Design of Controllers

A suitable model of the real object for control with self – tuning controllers is an input – output model. This is a standard approach in self tuning controller area. Instead of often tedious construction of a model from the first principles and then calculating its parameters from plant dimensions and physical constants, general type of model is chosen and its parameters are identified from data. Advantages of this kind of model are its simplicity and accuracy in an operational range in which the input – output dependence is measured. A model of the controlled system is then supposed to be expressed by a transfer function

$$G(q) = \frac{b(q)}{a(q)} \tag{1}$$

where $q = s$ for a continuous - time system and $q = z^{-1}$ for a discrete system

A model of the second order which is widely applied in practice and has proved to be effective for control of a range of various processes was chosen for description of dynamic behaviour of the further described system of interconnected tanks in the neighbourhood of a steady state. The polynomials a and b have the following forms in case of the continuous-time model

$$a(s) = s^2 + a_1 s + a_0,\; b(s) = b_0 \tag{2}$$

and

$$a(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2},\; b(z^{-1}) = b_1 z^{-1} + b_2 z^{-2} \tag{3}$$

in case of the discrete model. In both cases the polynomials a and b are supposed to be coprime.

Differential equation of the continuous – time system is given by

$$y'' + a_1 y' + a_0 y = b_0 u \tag{4}$$

and difference equation of the discrete system by

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) + b_1 u(k-1) + b_2 u(k-2) \tag{5}$$

The 2dof configuration of the closed loop with both feedback and feedforward parts was chosen. It is depicted in Fig. 1.

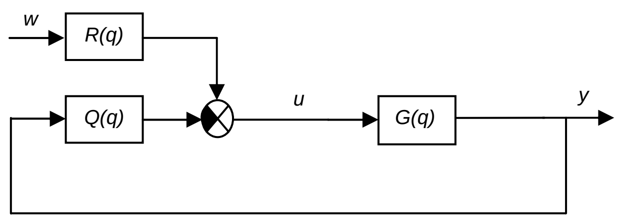

Fig. 1. The 2dof control system configuration.

The basic general requirements on the control system are:

- Internal properness and stability of the control system
- Asymptotic tracking of a reference

The controller has two parts: a feedback part and a feed-forward part. The feedback part is defined as

$$Q(q) = \frac{q(q)}{p(q)} \tag{6}$$

and the feed – forward part as

$$R(q) = \frac{r(q)}{p(q)} \quad (7)$$

Where q,p and r,p are coprime polynomials.

The reference signal can be also described as a ratio of two polynomials

$$W(q) = \frac{h_w(q)}{f_w(q)}, \deg h_w \le \deg f_w \quad (8)$$

For particular signals in the control loop can be derived following expressions in the complex domain (operator q will be omitted from some expressions for the purpose of simplification):

Controlled variable:

$$Y = \frac{1}{ap+bq}\left[br\frac{h_w}{f_w}\right] \quad (9)$$

Control error:

$$E = \frac{1}{ap+bq}\left[(d-br)\frac{h_w}{f_w}\right] \quad (10)$$

Manipulated variable:

$$U = \frac{r}{p}\frac{h_w}{f_w}\left(1-\frac{qb}{ap+bq}\right) \quad (11)$$

Stability of the closed loop is ensured by the feedback controller Q by solving equation

$$d = ap+bq \quad (12)$$

where d is a stable desired polynomial. Its poles are the ruling factors for the behaviour of the closed loop system.

Internal properness is ensured when the following conditions are fulfilled: $\deg q \le \deg p$ $\deg r \le \deg p$

To fulfill the requirement on the asymptotic tracking the denominator of the reference signal must be eliminated from the expression for the permanent control error.

$$\lim_{s\to 0}\left[s\cdot E(s)\right] = \lim_{s\to 0}\left[s\frac{1}{d}\left[(d-br)\frac{h_w}{f_w}\right]\right] = 0 \quad (13)$$

From the expression (13) it is obvious that the adequate condition of the asymptotic tracking is divisibility of the polynomial $d - br$ by the polynomial f_w. The following Diophantine equation must be then fulfilled

$$d - br = t\,f_w \quad (14)$$

where t is an unknown additional polynomial.

The resulting controller is given by solution of two Diophantine equations:

$$a\,p + bq = d \quad t\,f_w + br = d \quad (15)$$

Further it is necessary to determine degrees of particular polynomials. Relations for their computation are given by the requirement so that number of unknown controller's parameters and number of algebraic equations resulting from the Diophantine equations are equal. The requirement on the properness of the controller must be also fulfilled.

$$\deg q = \deg a - 1 \quad \deg p \ge \deg a - 1 + k \quad (16)$$

where

$$\begin{aligned} & k = \deg f_w - \deg a \\ & k = 0 \text{ for } \deg f_w - \deg a \le 0 \\ & \deg r = \deg f_w - 1 \quad \deg t = 2\deg a - 1 - \deg f_w + k \\ & \deg d \ge 2\deg a - 1 + k \end{aligned} \quad (17)$$

2.1 Design of Continuous – Time Controller

The reference was considered to be from a class of step and ramp signals.

The ramp reference signal $w(t) = w_0 t$ has the Laplace transform in the form $W(s) = \frac{w_0}{s^2} = \frac{h_w}{f_w}$. $\deg f_w = 2$ and $k = \deg f_w - \deg a = 2 - 2 = 0 \Rightarrow k = 0$. Degrees of the particular polynomials are derived according to expressions (16) and (17).

$$\begin{aligned} & \deg q = \deg a - 1 = 2 - 1 = 1 \\ & k = \deg f_w - \deg a = 2 - 2 = 0 \\ & \deg p \ge \deg a - 1 + k = 2 - 1 + 0 = 1 \\ & \deg r = \deg f_w - 1 = 2 - 1 = 1 \\ & \deg t = 2\deg a - 1 - \deg f_w + k = 4 - 1 - 2 + 0 = 1 \\ & \deg d \ge 2\deg a - 1 + k = 4 - 1 + 0 = 3 \end{aligned} \quad (18)$$

The polynomials then take following forms:

$$\begin{aligned} & q(s) = q_0 + q_1 s \quad p(s) = p_0 + p_1 s \quad r(s) = r_0 + r_1 s \\ & t(s) = t_0 + t_1 s \quad d(s) = s^3 + d_2 s^2 + d_1 s + d_0 \end{aligned} \quad (19)$$

The Diophantine equations (15) define sets of algebraic equations with unknown controller's parameters

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & a_1 & 0 & 0 \\ a_1 & a_0 & 0 & b_0 \\ a_0 & 0 & b_0 & 0 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ q_0 \\ q_1 \end{bmatrix} = \begin{bmatrix} 1 \\ d_2 \\ d_1 \\ d_0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & b_0 & 0 & 0 \\ b_0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r_0 \\ r_1 \\ t_0 \\ t_1 \end{bmatrix} = \begin{bmatrix} 1 \\ d_2 \\ d_1 \\ d_0 \end{bmatrix} \quad (20)$$

The parameters are obtained by solving these equations. The resulting controllers then take the form

$$Q(s) = \frac{q(s)}{p(s)} = \frac{q_1 s + q_0}{p_1 s + p_0} \qquad R(s) = \frac{r(s)}{p(s)} = \frac{r_1 s + r_0}{p_1 s + p_0} \quad (21)$$

The control law can be expressed by following equation

$$u' p_1 + u p_0 = r_1 w' + r_0 w - q_1 y' - q_0 y \quad (22)$$

2.2 Design of Discrete Controller

When The reference signal has Z – transform in the form $W(z) = \frac{w_0 T_v z}{(z-1)^2} = \frac{h_w}{f_w}$ where T_v is a sampling period. Degrees of the polynomials are computed according to expressions given in the previous section and the polynomials can be expressed as

$$\begin{gathered} p(z^{-1}) = 1 + p_1 z^{-1} \quad q(z^{-1}) = q_0 + q_1 z^{-1} \quad r(z^{-1}) = r_0 + r_1 z^{-1} \\ t(z^{-1}) = 1 + t_1 z^{-1} \quad d(z^{-1}) = 1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3} \end{gathered} \quad (23)$$

The sets of algebraic equations with unknown controller's parameters are as follows

$$\begin{bmatrix} 1 & b_1 & 0 \\ a_1 & b_2 & b_1 \\ a_2 & 0 & b_2 \end{bmatrix} \begin{bmatrix} p_1 \\ q_0 \\ q_1 \end{bmatrix} = \begin{bmatrix} d_1 - a_1 \\ d_2 - a_2 \\ d_3 \end{bmatrix} \begin{bmatrix} 1 & b_1 & 0 \\ -2 & b_2 & b_1 \\ 1 & 0 & b_2 \end{bmatrix} \begin{bmatrix} t_1 \\ r_0 \\ r_1 \end{bmatrix} = \begin{bmatrix} d_1 + 2 \\ d_2 - 1 \\ d_3 \end{bmatrix} \quad (24)$$

The controllers are defined as

$$Q(z^{-1}) = \frac{q(z^{-1})}{p(z^{-1})} = \frac{q_0 + q_1 z^{-1}}{1 + p_1 z^{-1}} \quad R(z^{-1}) = \frac{r(z^{-1})}{p(z^{-1})} = \frac{r_0 + r_1 z^{-1}}{1 + p_1 z^{-1}} \quad (25)$$

The control law takes the form

$$u(k) = r_0 w(k) + r_1 w(k-1) + q_0 y(k) + q_1 y(k-1) - p_1 u(k-1) \quad (26)$$

3 System Identification

3.1 Identification of Discrete Model

Various discrete linear models are used to describe dynamic behaviour of controlled systems. Overview of these models is given for example in [14]. The most widely applied linear dynamic model is the ARX model. Usually the ARX model is tested first and only if it does not perform satisfactorily more complex model structures are examined. But the ARX model matches the structure of many real processes. The parameters can be easily estimated by a linear least squares technique. Let the recursive least squares method was used as the basis of our algorithm. We can consider a stochastic process described by an ARX model in the form

$$a(z^{-1}) y(k) = b(z^{-1}) u(k) + e_s(k) \quad (27)$$

which can be expressed in a vector form

$$y(k) = \boldsymbol{\Theta}^T(k) \boldsymbol{\varphi}(k-1) + e_s(k) \quad (28)$$

where e_s is an un-measurable random component.

Vector of parameters is defined as

$$\boldsymbol{\Theta}^T(k) = [a_1, a_2, b_1, b_2,] \quad (29)$$

and the regression vector is

$$\boldsymbol{\varphi}^T(k-1) = [-y(k-1), -y(k-2), u(k-1), u(k-2)] \quad (30)$$

The aim of the identification is a recursive estimation of unknown model parameters $\boldsymbol{\Theta}$ on the basis of the inputs and the outputs considering the time moment k, $\{y(i), u(i), i = k, k-1, k-2, ..., k_0\}$ (where k_0 is an initial time of the identification). We are looking for a vector $\hat{\boldsymbol{\Theta}}$ minimizing the criterion

$$J_k(\boldsymbol{\Theta}) = \sum_{i=k_0}^{k} e_s^2(i) \quad (31)$$

where

$$e_s(i) = y(i) - \boldsymbol{\Theta}^T \phi(i) = \begin{bmatrix} 1 & -\boldsymbol{\Theta}^T \end{bmatrix} \begin{bmatrix} y(i) \\ \phi(i) \end{bmatrix} \quad (32)$$

Tracking of slow changes of the parameters, which is relevant owing to the control of the system of interconnected tanks (nonlinear system with variable parameters), can be achieved by application of exponential forgetting. This technique ensues from the assumption that new data describes the dynamics of an object better than older data, which are multiplied by smaller weighting coefficients. Then we minimize a modified criterion

$$J_k(\boldsymbol{\Theta}) = \sum_{i=k_0}^{k} \varphi^{2(k-i)} e_s^2(i) \quad (33)$$

where $0 \langle \varphi^2 \leq 1$ is the exponential forgetting factor.

In case that the identified plant is insufficiently activated – it means that the input and output signals are steady (this situation is typical for closed control systems), the exponential forgetting factor can cause numerical instability of the identification algorithm. A possible solution of this problem is application of the adaptive directional forgetting [15]. This technique changes the forgetting factor according to the level of information in the data. In the examples described bellow, the recursive least squares method supported by directional forgetting was applied.

In this case, the vector of parameters is actualised according to the following recursive expression

$$\hat{\boldsymbol{\Theta}}(k)=\hat{\boldsymbol{\Theta}}(k-1)+\frac{\boldsymbol{C}(k-1)\boldsymbol{\varphi}(k-1)}{1+\xi(k-1)}\hat{e}(k-1) \tag{34}$$

where

$$\xi(k)=\boldsymbol{\varphi}^T(k-1)\boldsymbol{C}(k-1)\boldsymbol{\varphi}(k-1) \tag{35}$$

is an auxiliary scalar and

$$\hat{e}(k)=y(k)-\hat{\boldsymbol{\Theta}}^T(k)\boldsymbol{\varphi}(k-1) \tag{36}$$

is a prediction error. If $\xi(t_k) > 0$, then the square covariance matrix $\boldsymbol{C}$ is actualised according to following expression

$$\boldsymbol{C}(k)=\boldsymbol{C}(k-1)-\frac{\boldsymbol{C}(k-1)\boldsymbol{\varphi}(k-1)\boldsymbol{\varphi}^T(k-1)\boldsymbol{C}(k-1)}{\varepsilon^{-1}(k)+\xi(k-1)} \tag{37}$$

where

$$\varepsilon(k)=\varphi(k)-\frac{1-\varphi(k)}{\xi(k-1)} \tag{38}$$

If $\xi(t_k)=0$, then

$$\boldsymbol{C}(k)=\boldsymbol{C}(k-1) \tag{39}$$

The directional forgetting factor is computed in each sampling period according to the expression

$$\varphi(k)=\left\{1+(1+\rho)[\ln(1+\xi(k-1))]+\left[\frac{(\upsilon(k-1)+1)\eta(k-1)}{1+\xi(k-1)+\eta(k-1)}-1\right]\frac{\xi(k-1)}{1+\xi(k-1)}\right\}^{-1} \tag{40}$$

where

$$\eta(k)=\frac{\hat{e}^2(k)}{\lambda(k)};\ \upsilon(k)=\varphi(k)[(\upsilon(k-1)+1];\quad \lambda(k)=\varphi(k)\left[\lambda(k-1)+\frac{\hat{e}^2(k-1)}{1+\xi(k-1)}\right] \tag{41}$$

are auxiliary variables.

3.2 Identification of Continuous -Time Model

It is not possible to measure directly input and output derivatives of a system in case of continuous – time control loop. One of the possible approaches to this problem is establishing of filters and filtered variables to substitute the primary variables. This approach is described in detail in [4], [5], [6]. The filtered variables are then used in the recursive identification procedure.

Let us consider a linear continuous – time ARX model in a form of differential equation

$$a(\sigma)y(t)=b(\sigma)u(t)+n(t) \tag{42}$$

where $n(t)$ is a random continuous – time variable and σ is the derivative operator. After the Laplace transform we obtain

$$a(s)Y(s)=b(s)U(s)+n(s)+o_1(s) \tag{43}$$

where the polynomial o_1 represents the Laplace transform of initial conditions. The output of the system is than given as

$$Y(s)=\frac{b(s)}{a(s)}U(s)+\frac{n(s)}{a(s)}+\frac{o_1(s)}{a(s)} \tag{44}$$

In order to obtain approximations of derivatives of the continuous – time variables it is necessary to establish filters using differential equations

$$c(\sigma)u_f(t)=u(t)\quad c(\sigma)y_f(t)=y(t) \tag{45}$$

where $c(\sigma)$ is a stable polynomial and u_f is a filtered input and y_f is a filtered output. After the Laplace transform we obtain

$$c(s)U_f(s)=U(s)+o_2(s);\quad c(s)Y_f(s)=Y(s)+o_3(s) \tag{46}$$

where $o_2(s)$ is a polynomial of initial conditions for the filtered input and $o_3(s)$ is a polynomial of initial conditions for the filtered output. The degree of the polynomial c must be greater or equal to the degree of the polynomial a ($deg\ c(s) \geq deg\ a(s)$). It is profitable to choose $deg\ c(s) = deg\ a(s)$ (the lower is the degree of the polynomial c, the faster is the dynamics of the filter). Time constants of the filters must be lower than time constants of the plant. A right choice of the filter's constants makes convergence of the parameters faster.

After substitution of the filtered variables to the equation (43) we obtain

$$a\left[cY_f(s)-o_3\right]=b\left[cU_f-o_2\right]+N(s)+o_1 \tag{47}$$

After modification and substitution

$$aY_f(s)=bU_f(s)+\frac{o_1-bo_2+ao_3+N(s)}{c}\qquad o=\frac{o_1-bo_2+ao_3}{c} \tag{48}$$

we obtain

$$Y_f(s)=\frac{b}{a}U_f(s)+\frac{o}{a}+\frac{1}{a}N(s)\quad\Rightarrow\quad G_f(s)=\frac{b}{a}=G(s) \tag{49}$$

Expression (49) proves that the transfer behaviour between the filtered and between the non – filtered variables is equivalent. Different are only initial conditions for the filtered and original variables. This fact enables to employ the filtered variables for the model parameter estimation.

After transformation to the time domain we obtain the following equation

$$a(\sigma)y_f(t) = b(\sigma)\,u_f(t) + n(t) \tag{50}$$

The filtered variables are taken in discrete time intervals $t_k = kT_s$, $k = 0,1,2, \ldots$, where T_s is the sampling period. The equation (50) can be modified to the form suitable for the model parameters estimation

$$y^{(n)}{}_f(t_k) = -\sum_{i=0}^{n-1} a_i y_f^{(i)}(t_k) + \sum_{j=0}^{m} b_j u_f^{(j)}(t_k) + n(t_k) \tag{51}$$

The parameters of the model are estimated by the recursive method described in the previous section according to expressions (34) - (41). For the considered continuous – time model given by expressions (2) and (4) the equation (51) takes following form

$$y_f''(t_k) = -a_1 y_f'(t_k) - a_0 y_f(t_k) + b_0 u_f(t_k) + n(t_k) \tag{52}$$

The regression vector and the vector of parameters are

$$\boldsymbol{\varphi}^T(t_k) = \left[-y_f'(t_k), -y_f(t_k), u_f(t_k)\right] \tag{53}$$

$$\boldsymbol{\Theta}^T(k) = [a_1, a_0, b_0] \tag{54}$$

Considering the order of the system, the filters for both variables were chosen to have second order.

$$\begin{aligned} y_f''(t) + c_1 y_f'(t) + c_0 y_f(t) &= y(t) \\ u_f''(t) + c_1 u_f'(t) + c_0 u_f(t) &= u(t) \end{aligned} \tag{55}$$

A right choice of the coefficients of the filter's polynomials and choice of the sampling period are the ruling factors for the speed of the parameter's convergence. Time constants of the filters must be lower than time constants of the plant.

4 Experimental Examples

4.1 Model of Interconnected Tanks

The controllers were verified and compared by real – time control of the model of interconnected tanks. Its principal scheme is on the following figure.

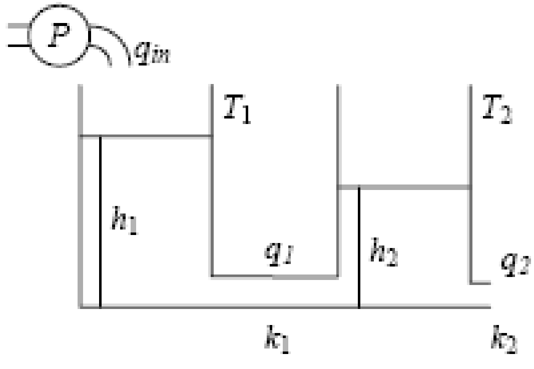

Fig. 2. Scheme of two interconnected tanks.

The system consists of two interconnected cylindrical tanks T_1 and T_2 and a pump P which induces inflow to the tank T_1. The liquid level heights in the tanks T_1 and T_2 are h_1 and h_2 respectively. The inflow produced by the pump is q_{in}, flow between tanks is q_1 and the outflow is q_2. The system can be considered as a single input single output system (SISO) where the input is inflow q_{in} and output is liquid level h_2.

The apparatus is a nonlinear system with variable parameters. The nonlinear behaviour is caused by characteristics of the valves, pipes and pumps. Additional nonlinearities are due to air bubbles which are present in the pipes and valves. The bubbles deflate from the pipe system in certain moments. Nonlinear mathematical model of the system of interconnected tanks is given in [16]. Its description by a linear model is then valid only in a neighbourhood of a steady state. As it was stated in the section 1, self-tuning controllers are a possible approach to the control of this kind of system. The nonlinear dynamics are described by a linear model in the neighbourhood of a steady state.

4.2 Experimental Results

In case of the continuous-time controller the sampling period for system identification (actualization of the parameters and transposition of the controller) was experimentally assigned as T_a=2s. The sampling period for actualization of the manipulated variable was assigned T_{v1}=0,02s. The polynomial d resulted from experiments in the form

$$d(s) = s^3 - 2{,}1s^2 + 1{,}47s - 0{,}343 \tag{56}$$

Constants of the filters were chosen to be c_0=0,04 c_1=0,4.

Best sampling period for discrete system was found as T_v=2s. The polynomial d was chosen as

$$d(z^{-1}) = 1 - 1{,}1805z^{-1} - 0{,}6390z^{-2} + 0{,}8195z^{-3} \tag{57}$$

Time responses of the control and identified parameters for step and ramp reference signals are in Fig.3 - Fig. 6.

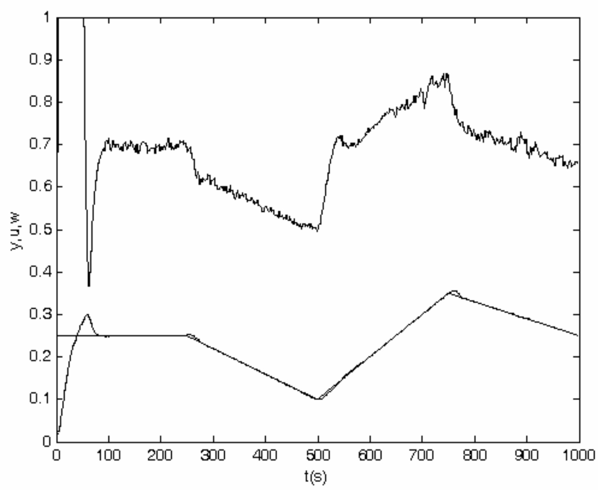

Fig. 3. Control with discrete controller.

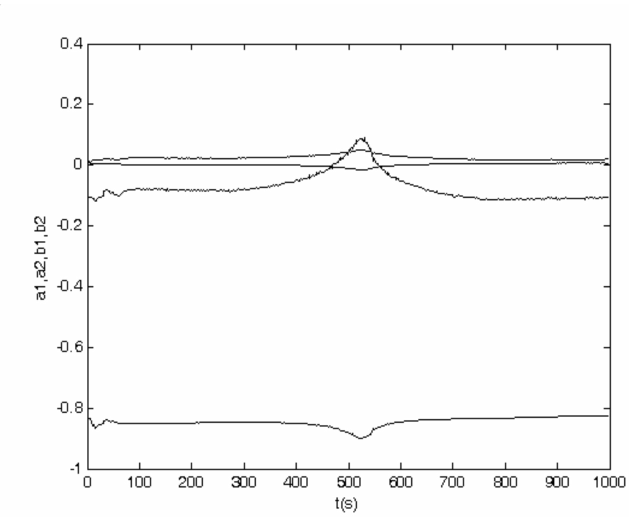

Fig. 4. Control with discrete controller – identified parameters.

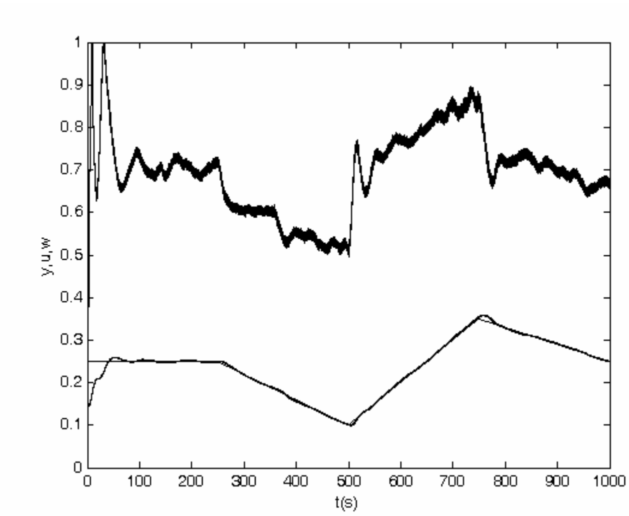

Fig. 5. Control with continuous - time controller.

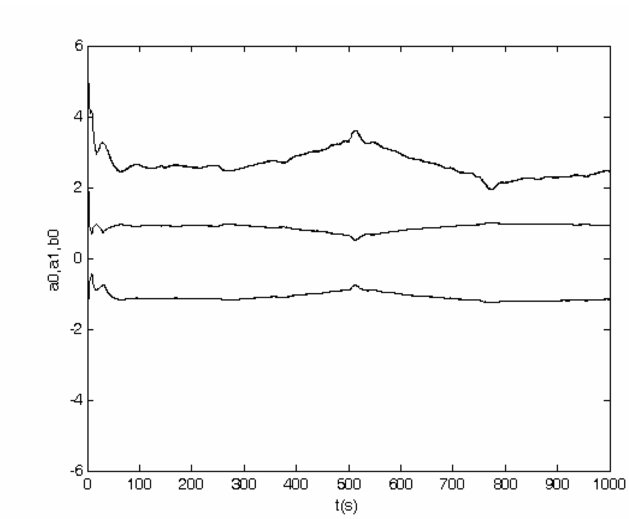

Fig. 6. Control with continuous - time controller – identified parameters.

4.3 Comparison of Control Performance Using Summing Criteria

The performances of both controllers were compared by means of control quality criteria, which are the sum of powers of tracking errors and the sum of increments of manipulated variables (both sums are related to the number of samples). In case of the continuous – time control the values were taken in discrete time intervals corresponding to sampling of the discrete control (T_v=2s). The Table 1 contains values of the criteria for the entire period of the experiments including the initial part of the parameters estimation. The Table 2 contains values obtained after the parameters fixation.

Table 1. Control quality criteria

Controller	$\sum e^2$	$\sum \Delta u^2$
Continuous - time	0,1396	0,2002
Discrete	0,7928	0,4254

Table 2. Control quality criteria – after parameters fixation

Controller	$\sum e^2$	$\sum \Delta u^2$
Continuous - time	0,0032	0,0410
Discrete	0,0078	0,0444

5 Extension for Two Input/Two Output Systems

Verification of the method by simulation was also carried out on a system with quite complex and complicated structure, a two – input/two – output system. This case was chosen in order to verify the algorithm on a system with more complex and complicated structure, where one output is influenced by more inputs. Most technological processes in practice are multivariable processes. They require that several variables relating to one system are controlled simultaneously. Each input may influence all system outputs. The two – input/two – output (TITO) processes are the most often encountered multivariable processes in practice and many processes with inputs/outputs beyond two can be treated as several TITO subsystems [17].

5.1 System Description

Internal structure of a two – input/two – output system with significant cross – coupling is shown in Fig. 7.

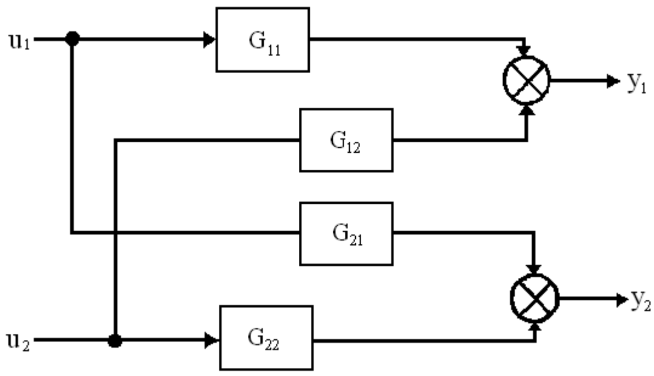

Fig. 7. Internal structure of a two – input/two output system with significant cross – coupling.

A general transfer matrix of a two – input/two – output system with significant cross - coupling between the control loops is expressed as

$$\boldsymbol{Y}(s)=\boldsymbol{G}(s)\boldsymbol{U}(s)=\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}\boldsymbol{U}(s) \tag{58}$$

where

$$\boldsymbol{U}(s)=[u_1(s),u_2(s)]^T \tag{59}$$

is the vector of manipulated variables and

$$\boldsymbol{Y}(s)=[y_1(s),y_2(s)]^T \tag{60}$$

is the output vector.

It is possible to assume that the dynamic behaviour of a two – input/two – output system can be described by a linear model in the form of the matrix fraction

$$\boldsymbol{G}(s)=\boldsymbol{A}^{-1}(s)\boldsymbol{B}(s)=\boldsymbol{B}_1(s)\boldsymbol{A}_1^{-1}(s) \tag{61}$$

where polynomial matrices $\boldsymbol{A}\in R_{22}[s]$, $\boldsymbol{B}\in R_{22}[s]$ are the left coprime factorization of the transfer matrix $\boldsymbol{G}(s)$ and matrices $\boldsymbol{A_1}\in R_{22}[s]$, $\boldsymbol{B_1}\in R_{22}[s]$ are the right coprime factorization of $\boldsymbol{G}(s)$. $\boldsymbol{U}(s)$ is a vector of manipulated variables and $\boldsymbol{Y}(s)$ is an output vector. As an example a model with polynomials of second order was chosen. This model proved to be effective for control of several TITO laboratory processes [18], where controllers based on a model with polynomials of the first order failed. The model has sixteen parameters. The matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are defined as follows

$$\boldsymbol{A}(s)=\begin{bmatrix} s^2+a_1s+a_2 & a_3s+a_4 \\ a_5s+a_6 & s^2+a_7s+a_8 \end{bmatrix} \tag{62}$$

$$\boldsymbol{B}(s)=\begin{bmatrix} b_1s+b_2 & b_3s+b_4 \\ b_5s+b_6 & b_7s+b_8 \end{bmatrix} \tag{63}$$

The differential equations of the model then take the form

$$\begin{aligned} &y_1''(t)+a_1y_1'(t)+a_2y_1(t)+a_3y_2'(t)+a_4y_2(t)= \\ &=b_1u_1'(t)+b_2u_1(t)+b_3u_2'(t)+b_4u_2(t) \\ &y_2''(t)+a_5y_1'(t)+a_6y_1(t)+a_7y_2'(t)+a_8y_2(t)= \\ &=b_5u_1'(t)+b_6u_1(t)+b_7u_2'(t)+b_8u_2(t) \end{aligned} \tag{64}$$

The equations analogical to the equation (51), which are suitable for the recursive model parameters estimation, are as follows

$$\begin{aligned} &y_{1f}''(t_k)=-a_1y_{1f}'(t_k)-a_2y_{1f}(t_k)-a_3y_{2f}'(t_k)-a_4y_{2f}(t_k)+ \\ &+b_1u_{1f}'(t_k)+b_2u_{1f}(t_k)+b_3u_{2f}'(t_k)+b_4u_{2f}(t_k)+\varepsilon_1(t_k) \\ &y_{2f}''(t_k)=-a_5y_{1f}'(t_k)-a_6y_{1f}(t_k)-a_7y_{2f}'(t_k)-a_8y_{2f}(t_k)+ \\ &+b_5u_{1f}'(t_k)+b_6u_{1f}(t_k)+b_7u_{2f}'(t_k)+b_8u_{2f}(t_k)+\varepsilon_2(t_k) \end{aligned} \tag{65}$$

The regression vectors have the form

$$\begin{aligned} &\phi_{1,2}{}^T(t_k)=[-y_{1f}'(t_k),-y_{1f}(t_k),-y_{2f}'(t_k),-y_{2f}(t_k), \\ &-u_{1f}'(t_k),-u_{1f}(t_k),-u_{2f}'(t_k),-u_{2f}(t_k)] \end{aligned} \tag{66}$$

and the parameter vectors are

$$\begin{aligned} &\boldsymbol{\Theta}_1{}^T(t_k)=[a_1,a_2,a_3,a_4,b_1,b_2,b_3,b_4] \\ &\boldsymbol{\Theta}_2{}^T(t_k)=[a_5,a_6,a_7,a_8,b_5,b_6,b_7,b_8] \end{aligned} \tag{67}$$

Considering the order of the system, the filters for all variables were chosen to have second order.

$$\begin{aligned} &y_{1f}''(t)+c_1y_{1f}'(t)+c_0y_{1f}(t)=y_1(t) \\ &y_{2f}''(t)+c_1y_{2f}'(t)+c_0y_{2f}(t)=y_2(t) \\ &u_{1f}''(t)+c_1u_{1f}'(t)+c_0u_{1f}(t)=u_1(t) \\ &u_{2f}''(t)+c_1u_{2f}'(t)+c_0u_{2f}(t)=u_2(t) \end{aligned} \tag{68}$$

5.2 Simulation Example

The structure of the model is given by the matrices (62) and (63). Identification of the following plant is given here as an example.

$$\boldsymbol{A}(s)=\begin{bmatrix} s^2+2s+0{,}7 & 0{,}2s+0{,}4 \\ -0{,}5s-0{,}1 & s^2+1{,}5s+0{,}3 \end{bmatrix} \tag{69}$$

$$\boldsymbol{B}(s)=\begin{bmatrix} 0{,}5s+0{,}2 & 0{,}1s+0{,}3 \\ 0{,}5s+0{,}1 & 0{,}3s+0{,}4 \end{bmatrix} \tag{70}$$

Fig. 8 shows the plant‘s step response

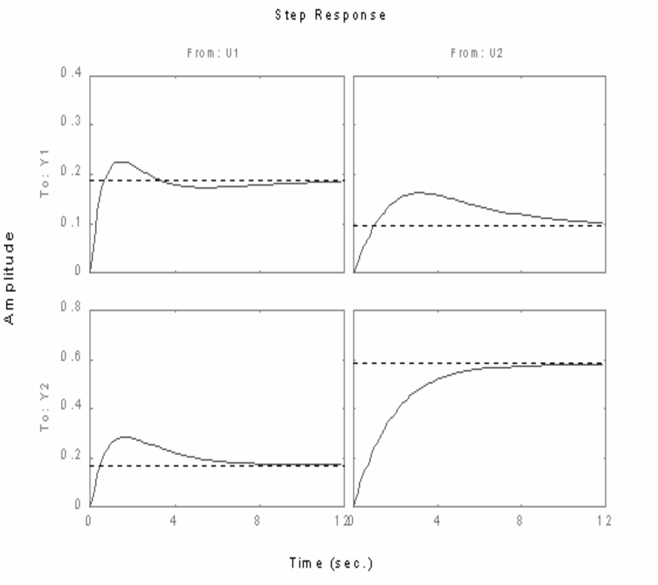

Fig. 8. Step response of the plant.

The system was activated by band limited white noise using zero – order – hold with sampling period $T_s = 1$.

The initial parameter estimates were chosen without any prior information

$$\boldsymbol{\Theta}_1^T(0) = [0.1, 0.2, 0.3, 0.4, 0.1, 0.2, 0.3, 0.4]$$

$$\boldsymbol{\Theta}_2^T(0) = [0.5, 0.6, 0.7, 0.8, 0.5, 0.6, 0.7, 0.8] \tag{71}$$

The filter parameters were experimentally chosen to be c_0=0,01 c_1=0,1. The continuous - time model parameters were recursively estimated in discrete time intervals t_k=kT_s, with the sampling period T_s=0,5. The sampling period was also assigned experimentally.

The trajectories of the identified parameters are shown in Fig. 9.

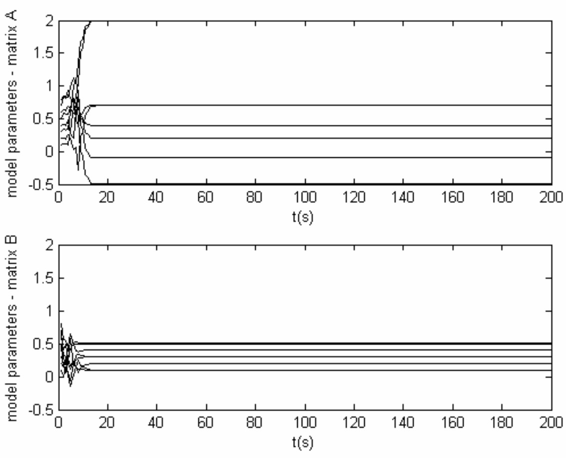

Fig. 9. Identified parameters.

The convergence of the parameters was satisfactory. The identified parameters converged to the parameters of the nominal system quickly.

5.3 Application to Multivariable Adaptive Control

The 2dof control configuration with two feedbacks was chosen. Design of multivariable controllers where systems are described by polynomial matrices is more effective with this controller structure than with the structure described for SISO systems in the section 2. This configuration was presented in [19].

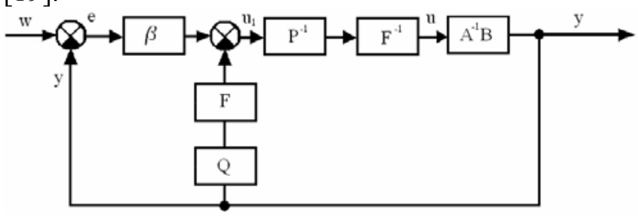

Fig. 10. Block diagram of the 2dof control configuration.

Generally, the vector $\boldsymbol{W}(s)$ of input reference signals is specified as

$$\boldsymbol{W}(s) = \boldsymbol{F}_w^{-1}(s)\boldsymbol{h}(s) \tag{72}$$

The reference signals are considered to be from the class of step functions. In this case $\boldsymbol{h}(s)$ is a vector of constants and $\boldsymbol{F_w}(s)$ is expressed as

$$\boldsymbol{F}_w(s) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \tag{73}$$

The compensator $\boldsymbol{F}(s)$ is a component formally separated from the controller. It has to be included in the controller to fulfill the requirement on the asymptotic tracking. If the reference signals are from the class of the step functions, then $\boldsymbol{F}(s)$ is an integrator.

It is possible to derive the following equation for the system output (operator s will be omitted from some equations for the purpose of simplification)

$$\boldsymbol{Y} = \boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{U} = \boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{F}^{-1}\boldsymbol{P}^{-1}\boldsymbol{U}_1 \tag{74}$$

where

$$\boldsymbol{U}_1 = \boldsymbol{\beta}(\boldsymbol{W} - \boldsymbol{Y}) - \boldsymbol{Q}\boldsymbol{F}\boldsymbol{Y} \tag{75}$$

The corresponding equation for the controller's output, as shown in the block diagram in Fig. 4, follows as

$$\boldsymbol{U} = \boldsymbol{F}^{-1}\boldsymbol{P}^{-1}\boldsymbol{U}_1 \tag{76}$$

The substitution of $\boldsymbol{U}_1$ and $\boldsymbol{Y}$ results in

$$\boldsymbol{U} = \boldsymbol{F}^{-1}\boldsymbol{P}^{-1}\left[\boldsymbol{\beta}(\boldsymbol{W} - \boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{U}) - \boldsymbol{Q}\boldsymbol{F}\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{U}\right] \tag{77}$$

Equation (77) can be modified using the right matrix fraction of the controlled system into the form

$$\boldsymbol{U} = \boldsymbol{A}_1\left[\boldsymbol{PFA}_1 + (\boldsymbol{\beta} + \boldsymbol{FQ})\boldsymbol{B}_1\right]^{-1}\boldsymbol{\beta W} \tag{78}$$

It is apparent, that the elements of the vector (78) have in their denominators the determinant of the matrix $\boldsymbol{PFA}_1$+($\boldsymbol{\beta}$+$\boldsymbol{FQ}$)$\boldsymbol{B}_1$. This determinant is the characteristic polynomial of a MIMO (multi – input/multi – output) system. The roots of this polynomial matrix are the ruling factors for the behaviour of a closed loop system. As for continuous – time systems, the roots must be in the left side of the Gauss complex plain in order for the system to be stable. The conditions for BIBO (bounded input bounded output) stability can be defined by the following diophantine equation

$$\boldsymbol{PFA}_1 + (\boldsymbol{\beta} + \boldsymbol{FQ})\boldsymbol{B}_1 = \boldsymbol{M} \tag{79}$$

Where $\boldsymbol{M} \in R_{22}[s]$ is a stable diagonal polynomial matrix

$$\boldsymbol{M}(s) = \begin{bmatrix} s^4 + m_1 s^3 + m_2 s^2 + \\ + m_3 s + m_4 & 0 \\ 0 & s^4 + m_5 s^3 + m_6 s^2 + \\ & + m_7 s + m_8 \end{bmatrix} \tag{80}$$

The degree of the controller polynomial matrices depends on the internal properness of the closed loop. The structure of matrices $\boldsymbol{P}$, $\boldsymbol{Q}$ and $\boldsymbol{\beta}$ was chosen so that the number of unknown controller parameters equals the number of algebraic equations resulting from the solution of the diophantine equation (79).

$$\boldsymbol{P}(s) = \begin{bmatrix} s + p_1 & p_2 \\ p_3 & s + p_4 \end{bmatrix} \tag{81}$$

$$\boldsymbol{Q}(s) = \begin{bmatrix} q_1 s + q_2 & q_3 s + q_4 \\ q_5 s + q_6 & q_7 s + q_8 \end{bmatrix} \tag{82}$$

$$\boldsymbol{\beta}(s) = \begin{bmatrix} \beta_1 & \beta_2 \\ \beta_3 & \beta_4 \end{bmatrix} \tag{83}$$

In both cases the solution of the diophantine equation results in a set of sixteen algebraic equations with unknown controller parameters. The controller parameters are obtained by solving these equations.

The control law apparent from the block diagram can be defined by matrix equation

$$\boldsymbol{FPU} = \boldsymbol{\beta E} - \boldsymbol{FQY} \tag{84}$$

The control law of the continuous – time controller in the form of differential equations is given by

$$\begin{aligned} u_1'' + p_1 u_1' &= \beta_1 e_1 + \beta_2 e_2 - q_1 y_1'' - \\ &- q_2 y_1' - q_3 y_2'' - q_4 y_2' - p_2 u_2' \end{aligned} \tag{85}$$

$$\begin{aligned} u_2'' + p_4 u_2' &= \beta_3 e_1 + \beta_4 e_2 - q_5 y_1'' - \\ &- q_6 y_1' - q_7 y_2'' - q_8 y_2' - p_3 u_1' \end{aligned} \tag{86}$$

5.4 Simulation Results

The controller described in the previous section was incorporated into an adaptive control system as the self – tuning controller, where the described identification procedure was applied.

Control of the system described in section 5.2 is given here as an example. The matrix $\boldsymbol{M}(s)$ on the right side of the diophantine equation (79) obtained from experiments was

$$\boldsymbol{M}(s) = \begin{bmatrix} s^4 + 4s^3 + 6s^2 + \\ + 4s + 1.5 & 0 \\ 0 & s^4 + 4s^3 + 6s^2 + \\ & + 4s + 1.5 \end{bmatrix} \tag{87}$$

The initial parameter estimates were again chosen without any prior information (71).

The filter parameters and the sampling period were chosen to be the same as it was in the previous simulation example (c_0=0,01 c_1=0,1 T_s=0,5). The model parameters were recursively estimated in discrete time intervals given by the sampling period. The time responses of the control are shown in Figs 11 – 12 and the courses of the identified parameters are in Fig. 13.

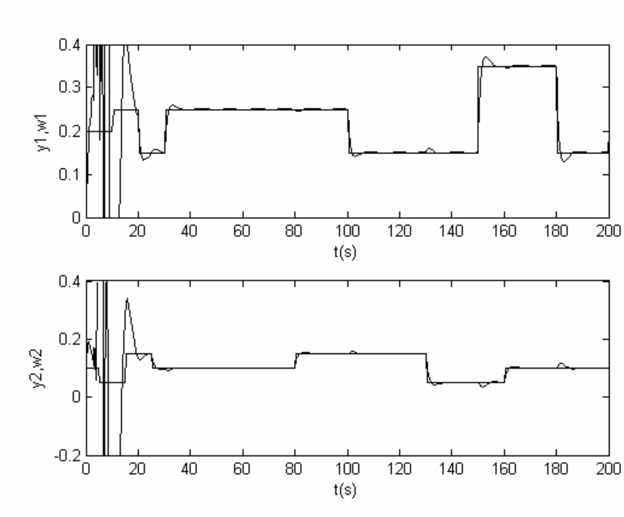

Fig. 11. Control with continuous – time controller

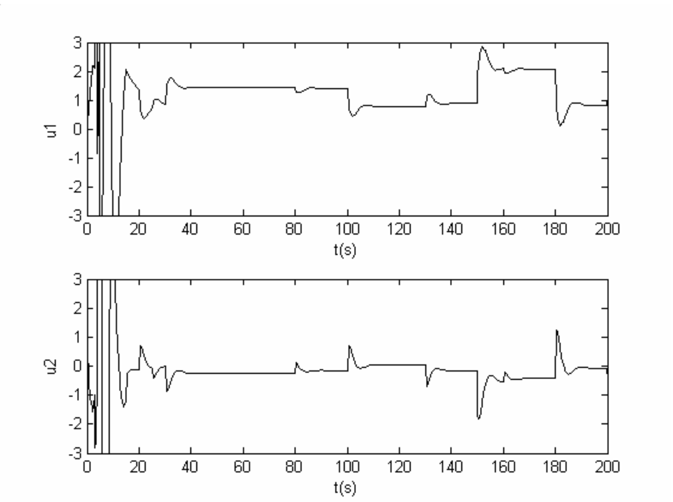

Fig. 12. Control with continuous – time controller – manipulated variables

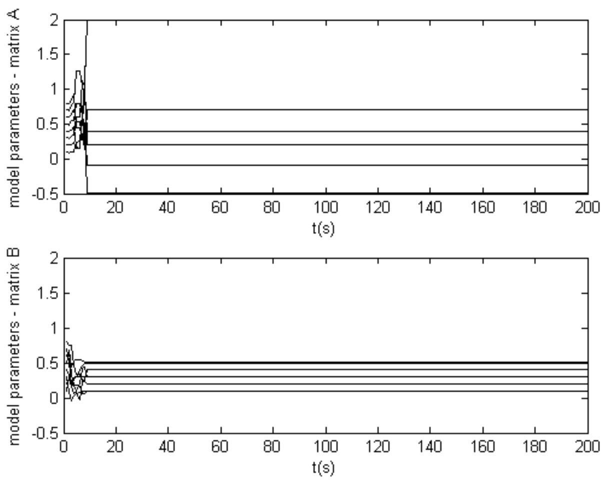

Fig. 13. Identified parameters

Convergence of the identified parameters was again good and fast.

6 Conclusion

The continuous – time adaptive controller was verified by simulation and it was implemented for control of liquid level of the interconnected tanks. The continuous - time model of the controlled system was recursively estimated and design of the controller was performed in the continuous – time domain.

The main advantage of the approach is that it enables fast sampling. The value of the sampling period is then dependant only on capabilities of the used hardware and software. The used software must enable realization of filters by differential equations.

The applied controller was compared to an analogical discrete controller. Performances of the discrete and continuous – time controllers were comparable. According to the chosen control quality criteria (Tables 1 and 2) slightly better performed the continuous - time controller. On the other hand, experimental tuning of the continuous – time controller was more complicated. It was rather difficult to find experimentally a suitable conjunction of the tuning parameters. Adjustable parameters are poles of the characteristic polynomial, constants of the filters and the sampling periods T_a and T_{v1}. There is a lack of clear theory relating to the closed loop behavior to design parameters. Control courses were rather sensitive to changes of the parameters.

The described method of continuous – time models parameters estimation proved to be effective. A right choice of the filter's constants and the sampling period improves convergence of the parameters. The method is suitable for the identification part of continuous – time self – tuning controllers.

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