Modelling, identification and simulation of the inverted pendulum PS600

Jiří Marholt *and* ***František Gazdoš***[1]

This paper deals with modelling, identification and simulation of the inverted pendulum PS600. This system is unstable and nonlinear with one input – voltage of a DC motor which can change position of the cart and two outputs – cart position and angle of the pendulum rod. First, a mathematical model of the inverted pendulum is derived, followed by identification of its unknown parameters. Then, the created mathematical model is implemented into the MATLAB/Simulink environment and its properties are experimentally compared with the real system.

Key words: *modelling, identification, simulation, inverted pendulum PS600.*

Introduction

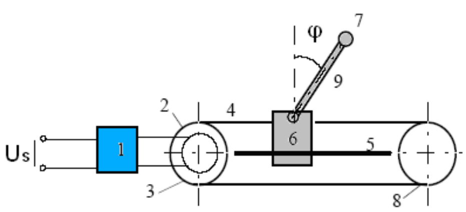
There are a lot of processes in industry that possess instable behaviour, such as various types of reactors, combustion systems, distillation columns, etc (Padma Sree, Chidambaram, 2006). Real-time experiments with these systems without proper control can be a real hazard (Stein, 2003). In such cases, process modelling and simulation are often the only safe tools to investigate properties of such systems. Nowadays, the role of modelling and simulation has risen mainly due to the increasing performance of computer technology. There are plenty of sources devoted to these science areas, e.g. (Wellstead, 1979; Severance, 2001; Mikleš & Fikar, 2007).

This contribution is focused on modelling and simulation of the inverted pendulum PS600 which represents one type of unstable systems. The real system is a product of the AMIRA company and can be found in laboratories of Department of Process Control, Faculty of Applied Informatics, Tomas Bata University in Zlín. The goal of this work is to develop a reliable mathematical model of the system together with its linearized version and corresponding simulation environment which will help both students and researchers to experiment with this unstable system. The resultant model is then implemented into the MATLAB/Simulink environment, e.g. (Hanselman & Littlefield, 2005; Dabney & Harman, 2004).

The paper is structured as follows: after the introduction a description of the system follows together with a proposed mathematical model and its parameters. Next, linearization in chosen operating points is presented and resultant transfer functions are analyzed. Further, comparison of the proposed mathematical model with the real system is performed and finally some conclusions are stated.

Inverted pendulum

The system consists of a cart which can be moved along a metal guiding bar. An aluminium rod with a cylindrical weight is fixed to the cart by an axis. This system is unstable and non-linear with one input and two outputs. Input signal is control voltage of a DC motor which can change position of the cart. The outputs are cart position and angle of the pendulum rod. Both outputs are measured by incremental encoders. Scheme of the system is presented in fig. 1.



1 – servo amplifier,
2 – motor,
3 – drive wheel,
4 – transmission belt,
5 – metal guiding bar,
6 – cart,
7 – pendulum weight,
8 – guide roll,
9 – pendulum rod.

Fig. 1. Scheme of the inverted pendulum.

[1] *Jiří Marholt, František Gazdoš*, Department of Process Control, Faculty of Applied Informatics, Tomas Bata University in Zlín, Zlín, Czech Republic, jiri-83@seznam.cz
(Review and revised version 23. 2. 2010)

Mathematical model

The system can be described by following nonlinear differential equations (Amira, 2000),

$$m\,\ddot{r} + F_r\,\dot{r} + m_K\,l\,\ddot{\varphi}\cos\varphi - m_K\,l(\dot{\varphi})^2\sin\varphi = F \tag{1}$$

$$\Theta\,\ddot{\varphi} + C\,\dot{\varphi} + m_K l\,g\sin\varphi - m_K l\,\ddot{r}\cos\varphi = 0 \tag{2}$$

where F represents input signal, which is the force produced by the DC motor. Output signals are r – cart position, $\dot{r}$ – cart speed, φ – pendulum angle, $\dot{\varphi}$ – pendulum angular speed. Symbol g is the gravity acceleration constant and F_r represents constant of a velocity proportional friction of the cart. All constants and symbols are clearly defined in Table 1. Following substitutions were used in the equations (1) and (2):

$$\Theta = \Theta_S + m_K l^2 \tag{3}$$

$$m = m_V + m_K \tag{4}$$

where m_V is the cart weight, m_K is the pendulum weight, l is the distance between centre of gravity of the pendulum and the centre of rotation of the pendulum and Θ_s represents the inertia moment of the pendulum rod with respect to the centre of gravity.

All the used constants were either taken from the producer (Amira, 2000) or identified by experiments (Chalupa & Bobál, 2008; Marholt, 2008). Their symbols and values are clearly defined in Tab. 1.

Tab. 1. Parameters of the real system.

Parameter	Symbol	Value & Unit
weight of the cart	m_V	4 kg
weight of the pendulum	m_K	0,42 kg
total weight	m	4,42 kg
length of the pendulum	l	0,42 m
inertia moment	Θ	0,08433 kgm^2
velocity proportional friction of the cart	F_r	6,5 N
friction of the cart	F_C	15 N
friction of the pendulum	C	0,006 $kg\,m^2\,s^{-1}$
rate constant	k_A	7,5 N/V

Linearization

For the purpose of control design, nonlinear differential equations (1) and (2) describing behaviour of inverted pendulum were linearized in the operating points 0 (bottom position of the pendulum) and π (top position). Consequently, the following transfer functions were computed.

Transfer function of cart position for operating point 0 is in the form:

$$G^0_{r/F}(s) = \frac{0,2469s^2 + 0,0191s + 5,065}{s^4 + 1,689s^3 + 22,51s^2 + 32,93s}$$

Properties of the transfer function:

- integrating system
- poles: $p_1 = 0$, $p_2 = -1,4830$,
- $p_{3,4} = -0,1030 \pm 4,7110i$
- zeros: $n_{1,2} = -0,0387 \pm 4,5291i$

Therefore, besides the integrating pole p_1, the system is stable, periodic and minimum-phase.

Transfer function of pendulum angle for the operating point 0 takes the form:

$$G^0_{\varphi/F}(s) = \frac{-0,5164s}{s^3 + 1,689s^2 + 22,51s + 32,93}$$

Properties of the transfer function:

- derivative system
- poles: : $p_1 = -1,4830$, $p_{2,3} = -0,1030 \pm 4,7110i$

- zero: $n = 0$

  Hence, besides the derivative behaviour, the system is stable periodic.

  Transfer function of the cart position for the operating point π is in the form:

$$G^{\pi}_{r/F}(s) = \frac{0,2469s^2 + 0,0191s - 5,065}{s^4 + 1,689s^3 - 22,27s^2 - 32,93s}$$

Properties of the transfer function:

- Integrating system
- poles: $p_1 = 0$, $p_2 = 4,6400$,
- $p_3 = -1,4565$, $p_4 = -4,8725$
- zero: $n_1 = 4,4908$, $n_2 = -4,5681$

Therefore, besides the integrating behaviour, the system is unstable periodic and non-minimum phase.
Transfer function of the pendulum angle for the operating point π takes the form:

$$G^{\pi}_{\varphi/F}(s) = \frac{0,5164s}{s^3 + 1,689s^2 - 22,27s^1 - 32,93}$$

Properties of the transfer function:

- derivative system
- poles: $p_1 = 4,6400$, $p_2 = -1,4565$,
- $p_3 = -4,8725$
- zero: n = 0

Hence, besides the derivative behaviour, the system is unstable.

Simulation

With the help of the MATLAB/Simulink environment and the Real-time toolbox (Humusoft, 2003), behaviour of the inverted pendulum for selected values of the control signal was found. Then, the recorded data of the cart position and pendulum angle were compared with the data simulated using the proposed mathematical model.

In the following experiments, the control signal of the value -0,3 was used which influenced the system in the time-interval 4-5s. The following graphs fig. 2 and fig. 3 show courses of the mathematical and real model behaviour. Here, the individual symbols denote *rs* – control signal, p_n – real system cart position, p_s – mathematical model cart position, u_n – real system pendulum angle, u_n – mathematical model pendulum angle.

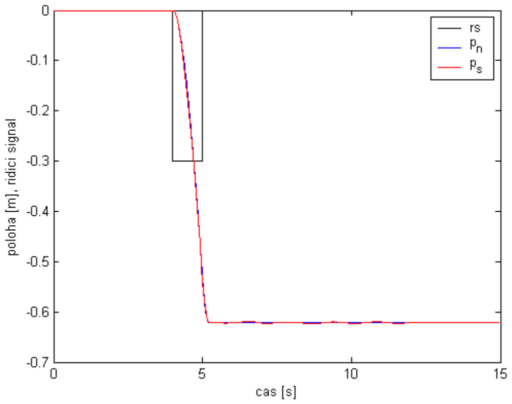


Fig. 2. Cart position comparison for the mathematical and real model.

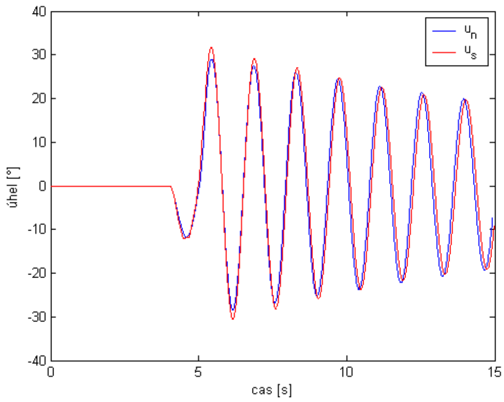


Fig. 3. Angular displacement of the pendulum for the mathematical and real model.

Further comparison was e.g. implemented using a PI controller during cart position control where the classical Hurwitz method for the closed-loop stability verification was used, e.g. (Dostál & Gazdoš, 2006). The resultant control response is presented in fig. 4.

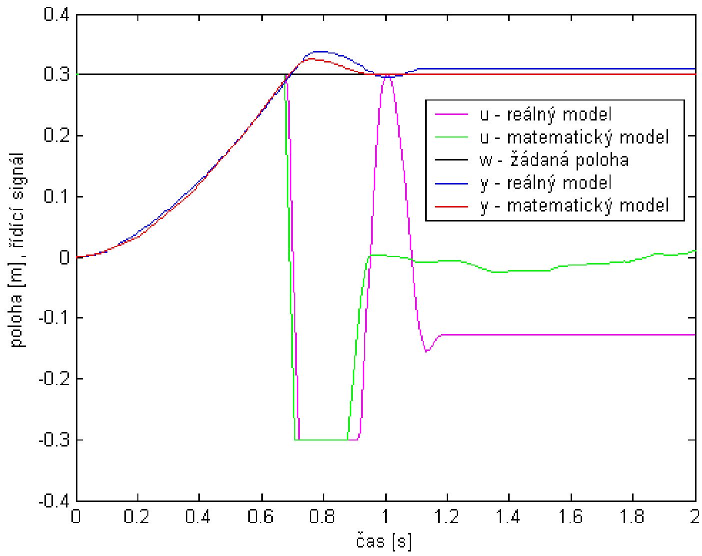


Fig. 4. Cart position control using a PI controller.

From the presented comparison we can conclude that the proposed mathematical model describes behaviour of the real-time system quite well, at least in terms of the controlled variable.

Conclusion

The aim of this paper was modelling, identification and simulation of the inverted pendulum PS600. The real prototype is located in the laboratory of the Department of Process Control, Faculty of Applied Informatics of the Tomas Bata University in Zlín The resultant nonlinear differential equations describing the behaviour of the system were implemented into the MATLAB/Simulink environment with the help of the special block “S-Function”. Properties of the proposed mathematical model were compared with its real counterpart. Resultant courses of the cart position and angular displacement of the inverted pendulum for the selected course of control signal is depicted in fig. 2-3.

Further presented comparison was done during cart position control by a conventional PI controller, see fig. 4. From the presented graphs it can be concluded that the dynamics of the proposed mathematical model corresponds to the real system quite well. As a result, the proposed model can be further used for both simulation analysis and synthesis of this system. In addition, the derived transfer functions of the inverted pendulum in different operating points can be further used for control design and experiments with this type of unstable systems.

Acknowledgement
This work was supported by the Grant Agency of the Czech Republic under the grant no. 102/07/P148.

References

Amira: PS600 Laboratory Experiment Inverted Pendulum. *Duisburg : Amira GmbH, 2000, 351 pp*.

Chalupa, P., Bobál, V.: Modelling and predictive control of inverted pendulum. In *22nd European Conf. Modelling and Simulation*. Nicosia, Cyprus : Digitaldruck Pirrot GmbH Sbr.-Dudweiler, *2008, pp. 531-537. ISBN 978-0-9553018-5-8*.

Dabney, J. B., Harman, T. L.: Mastering Simulink. *Prentice Hall, 2004. ISBN 0-13-142477-7*.

Dostál, P., Gazdoš, F.: Control of technological processes *Zlín : Tomas Bata University in Zlín, 2006. 98 pp. ISBN 80-7318-465-6*.

Hanselman, D. C., Littlefield, B.: Mastering Matlab 7. Prentice *Hall, 2005. ISBN 0-13-143018-1*.

Humusoft: Real time toolbox - user’s manual. *Prague : Humusoft s.r.o., 2003*.

Marholt, J.: Modelling, identification and simulation of the inverted pendulum PS600 (in Czech). *Diploma thesis. Zlín : Tomas Bata Univ. in Zlín, 2008, 75 pp*.

Mikleš, J., Fikar, M.: Process modelling, identification, and control. *Berlin : Springer-Verlag, 2007, 480 pp., ISBN 978-3-540-71969-4*.

Padma Sree, R., Chidambaram, M.: Control of unstable systems. *Oxford : Alpha Science Int. Ltd., 2006, 297 pp., ISBN 1-84265-287-7*.

Severance, F. L.: System modeling and simulation. *Chichester : Wiley, 2001, 518 pp., ISBN 0471496944*.

Stein, G.: Respect the unstable. *IEEE Control system Magazine*. *August 2003, pp. 12-25., 2003*.